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Labor market imperfections, real wage rigidities and financial shocks*

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Abstract
By using the recent Gertler and Kiyotaki’s (2010) setup, this paper explores the interaction between real distortions stemming from the labor market institutions and financial shocks. We find that neither labor market imperfections nor fiscal institutions determining tax wedges have an impact on the volatility of the real economy induced by a financial shock. By contrast, real wage rigidities matter as they amplify the financial shock effects. Thus, economies with larger imperfections will not systematically observe larger or smaller recessions, unless a causality between imperfections and real wage rigidities is introduced.

Jel codes: E32, E44.  
Keywords: Financial accelerator, credit frictions, wage-setters, business cycle, volatility.

1 Introduction
The idea that imperfections in the financial sphere amplify the business cycle is an old one. The notion of a financial accelerator by which large fluctuations in aggregate economic activity seem to arise from seemingly small shocks dates back to Fisher (1933), and has been fully formalized by Bernanke et al. (1996).1 Recently, after the eruption of the financial crisis, the possibility that adverse conditions in the real economy and in financial markets mutually reinforce each other has been revisited by a number of authors.2 A new wave of models of

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1Founding their model on Bernanke and Gertler (1989) and Kiyotaki and Moore (1997), who focus on the costs of borrowing and lending associated with asymmetric information, Bernanke et al. (1996) first introduced a formal model of financial accelerator in a DSGE model with consumers-lenders and firms-borrowers.

economies with financial frictions has emerged where a formal bank channel is specifically considered.\(^3\)

Our paper contributes to the above literature by jointly considering labor market and financial market frictions. The interactions between these two kinds of imperfections has been, in fact, scarcely studied. Wasmer and Weil (2004) and Hristov (2009) are an exception. Wasmer and Weil (2004) employ a simple macroeconomic model reminiscent of the IS-LM framework, with endogenous search frictions in both the credit and the labor market, thus extending to the former market the mechanism introduced by Pissarides (1988) for the latter. In their model a financial accelerator operates deriving from general equilibrium interaction between imperfections in the two markets. Hristov (2009), instead, makes use of a New Keynesian DSGE model with a search mechanism for the labor market and costly state veriﬁcation in the credit sector. Important as they are, these papers do not investigate on the whole set of labor frictions, as both make use of a search model for the labor market, which emphasizes some types of such frictions to the detriment of others.

Notwithstanding the recent success of search models in DSGE contexts,\(^4\) we prefer to follow a different approach, inspired by recent Blanchard and Galì’ s methodological suggestions\(^5\) to emphasize the role played by the labor wedge and real wage rigidities, which traditionally indicate the economic impact of the main labor market institutions (see, among others, Layard \textit{et al.}, 1991; Belot and Van Ours, 2001). Specifically, we consider a simple RBC model augmented by a financial accelerator mechanism, where a negative shock to the value of capital held by banks leads to a credit crunch, as a crunch is the only way to restore the proﬁtability of banks and therefore to ensure that bankers do not run away with the depositors’ funds (Gertler and Kiyotaki, 2010). To this scenario we add labor market frictions, namely the labor wedge and real wage rigidities. As is well known, the former is given by the difference between the marginal rate of substitution between consumption and leisure and the marginal productivity of labor. This difference is brought about by imperfect substitutability between different kinds of labor, market power of unions, as well as distortionary taxation both in the form of payroll and labor income taxes. The latter instead have to do with factors such as wage stickiness, duration of labor contracts, hiring and firing costs.

We find that only real wage rigidities interact with financial frictions, and in fact create a feedback loop that propagates the financial and macroeconomic downturn. By contrast, labor market imperfections inﬂuencing the labor wedge do not amplify the reaction of the economy to adverse shocks, i.e. they have an impact on steady state values but not on the dynamics of innovations. A sort of dichotomy emerges: institutions affecting the labor wedge only inﬂuence the economy in the long run, whereas real wage rigidities are only relevant for the business cycle. The result is not trivial as complementarities among the two types of labor market frictions cannot be neglected a priori.

The rest of the paper is organized as follows. Section 2 illustrates our model. Section 3 describes our main results and provides their intuition. Specifically,

\(^3\)It is worth noticing that the bank system in the original model of Bernanke and his coauthors was a sort of veil.

\(^4\)As said, in the financial accelerator literature the role of search in the labour and/or the financial market is emphasized by Wasmer and Weil (2004) and Hristov (2009).

we first show our result of neutrality for the labor market imperfections driven by the labor wedge with respect to the financial-shock-driven fluctuations and then we investigate the impact of real wage rigidities. A final section concludes.

2 The model

Our core framework is a real business cycle model with distorted labor and financial markets and real wage rigidities, based on Gertler and Kiyotaki (2010). We consider a simple setup assuming no idiosyncratic uncertainty for producing firms and homogeneous financial intermediaries. Households consist of both workers and bankers and perfect consumption insurance among them is guaranteed. We consider a non-competitive labor market with strategic wage setters and real wage rigidities. In this market, workers supply hours to non-financial firms and return wages to the household. Similarly, bankers transfer profits earned from the financial activity back to their family. Homogeneous banks intermediate funds between households and non-financial firms in the financial market, facing endogenously determined balance sheet constraints due to an agency problem. Banks provide funds against future profits of the firms which are able to offer perfect state contingent debt. Thus we can think of the banks’ claims as equities. Competitive non-financial firms produce output by means of capital and labor. Finally, competitive capital producing firms owned by the households are also introduced.

2.1 Households

In the economy there is a continuum of infinitely lived households indexed by $i$ on the unit interval $(0, 1)$; each of them supplies a differentiated labor type. Preferences of households are defined over consumption $(C_{t,i})$ and hours worked $(L_{t,i})$:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_{t,i}, L_{t,i}) = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln(C_{t,i} - hC_{t-1,i}) - \frac{\chi}{1 + \varepsilon} L_{t,i}^{1+\varepsilon} \right]$$

(1)

with $\beta \in (0, 1)$. $h$ is the habits in consumption parameter, $\chi$ measures the relative weight of the labor argument and $\varepsilon$ is the inverse Frisch elasticity of labor supply.

The household budget constraint at time $t$ is:

$$C_{t,i} = (1 - \tau_L) W_{t,i} L_{t,i} + \Pi_{t,i} + R_t D_{t-1,i} - D_{t,i} - T_t$$

(2)

where $D_{t-1,i}$ is the total quantity of short term debt the household acquires from banks or government in the form of real bonds that pay the gross real
return $R_t$ over the period from $t - 1$ to $t$; $W_{t;i}$ is the real wage, $Y_{t;i}$ is the net payout to the household from ownership of both non-financial and financial firms; $T_i$ is a lump sum tax; $\tau_L$ indicates the labor income tax rate.

Households first order conditions imply a standard Euler condition: \(^{10}\)

$$1 = \beta E_t \frac{U_{Ct+1}}{U_{Ct}} R_{t+1}$$

where $U_C$ is the marginal utility of consumption which is defined as follows:

$$U_{Ct} = \frac{1}{C_t - hC_{t-1}} - \frac{\beta h}{C_{t+1} - hC_t}$$

Thus, $\Lambda_{t,t+1} = \beta U_{Ct+1}/U_{Ct}$ is the household’s discount factor. The condition about the optimal labor supply will be introduced at a later stage, when we consider the labor market.

2.2 The Real Sector

2.2.1 Final good producing firms

The economy is populated by a continuum of symmetric competitive good producing firms indexed by $f$ on the unit interval $(0, 1)$; they employ both capital ($K_{t-1}$) and labor ($L_t$) as inputs. Each firm produces perfectly substitutable goods given a Cobb-Douglas production function:

$$Y_{t,f} = A_t K_{t-1,f}^{\alpha} L_t, f^{1-\alpha}$$

where $A_t = \exp(a_t)$ is an aggregate productivity shock, with $a_t = \rho_a a_{t-1} + u_t$, and $u_t$ a i.i.d. normal variable and $L_{t,f}$ denotes a labor bundle of imperfect substitutable labor types distributed over a unit interval, represented by:

$$L_{t,f} = \left[\int_0^1 L(i, f) \, di\right]^{\eta-1}$$

where $\eta$ is a measure of the wage setters’ monopoly power (i.e., the intra-temporal elasticity of substitution across different labor inputs).

For any given level of its labor demand, $L_{t,f}$, each firm must decide the optimal allocation across labor inputs, subject to the aggregation technology (6). From the minimization cost problem solution, demand for labor type $i$ by firm $f$ is then:

$$L(i)_{t,f} = \left(\frac{W_t(i)}{W_t}\right)^{-\eta} L_{t,f}$$

where

$$W_t = \left[\int_0^1 W_t(i)^{1-\eta} \, di\right]^{\frac{1}{1-\eta}}$$

is the average real wage index.

\(^9\)Note that $\Pi_{t;i}$ is net of the transfer the household gives to its members that enter banking at time $t$.

\(^{10}\)Index $i$ is dropped for simplicity.
Firms equate the marginal productivity of labor to the wage. As firms are symmetric we can just drop the index $f$ and obtain aggregate labor demand:

$$L_t = \left( \frac{(1 + \tau_S) W_t}{A_t K_{t-1}^\alpha (1 - \alpha)} \right)^{\frac{1}{\alpha}} \tag{9}$$

where $\tau_S$ is a payroll tax, or:

$$W_t = \frac{1 - \alpha}{1 + \tau_S} \frac{Y_t}{L_t} \tag{10}$$

As far as capital services demand is concerned, we observe that the gross profit per unit of capital $Z_t$ is given by:

$$Z_t = \frac{Y_t - (1 + \tau_S) W_t L_t}{K_t} = \alpha A_t \left( \frac{L_t}{K_t} \right)^{1-\alpha} \tag{11}$$

Firms are financed by banks, who collect the savings of households. Firms buy new capital goods from capital producers by issuing state-contingent equities at price $Q_t$ and committing to pay the flow of future gross capital profits to the banks.

### 2.2.2 Capital producing firms

There is a continuum of length one of competitive capital producing firms.\[^{11}\] They transform one unit of final good into one unit of capital good (priced $Q_t$) subject to a flow adjustment cost. Thus, the representative capital producing firm maximizes the following expected present discounted value of future profits:\[^{12}\]

$$E_t \sum_{\tau=0}^{\infty} \Lambda_{t, \tau+1} \left( (Q_t - 1) I_t - f \left( \frac{I_t}{I_{t-1}} \right) I_t \right)$$

where $I_t$ is the production (i.e., investment) and $f(\cdot)$ is the adjustment cost function. We assume that $f(1) = f'(1) = 0$ and $f''(I_t/I_{t-1}) > 0$; $f(I_t/I_{t-1}) I_t$ is physical adjustment costs.

Profit maximization implies:

$$Q_t = 1 + f \left( \frac{I_t}{I_{t-1}} \right) + \frac{I_t}{I_{t-1}} f' \left( \frac{I_t}{I_{t-1}} \right) - E_t \Lambda_{t, \tau+1} f'' \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \tag{12}$$

The law of motion for capital is given by:

$$K_{t+1} = \Psi_{t+1} (I_t + K_t (1 - \delta)) \tag{13}$$

where $\delta$ is the capital depreciation rate and $\Psi_t = \exp(\psi_t)$ is a capital quality shock, i.e., an exogenous source of variation in the value of capital; $\psi_t = \rho_0 \psi_{t-1} + \varepsilon_t$ and $\varepsilon_t$ is a i.i.d. normal variable with zero mean and finite variance, $\sigma^2$.

\[^{11}\]Firms’ indices are dropped for simplicity.

\[^{12}\]Capital producing firms earn no profits in steady state; when fluctuations occur they redistribute profits directly to the households who own capital producing firms.

\[^{13}\]See Gertler and Karadi (2009), Brunnermeier and Sannikov (2009) and Gourio (2009) for this kind of shock.
2.2.3 Labor markets

Differently from Gertler and Kiyotaki (2010), the labor market is not competitive as each worker sells a different kind of labor. Each wage-setter bargains over the real wage, taking other workers’ decisions as given. However, wage setting might be coordinated to various degrees. The coordination degree is captured by the parameter $n$ in the following way. Each wage-setter (indexed by $j$, with $j = 1, \ldots, n$) acts on behalf of a length $n^{-1}$ of workers. More specifically, each union $j$ sets the wage $W_{t,j}$ of the agent $i \in j$, (i.e., $W_{t,i} = W_{t,j}$ if $i \in j$) so as to maximize his utility in (1), subject to the budget constraint (2), (7) and (9).

In fact, by (8), in the decentralized equilibrium each union $j$ anticipates that

$$
\frac{\partial W_t}{\partial W_{t,j}} = \frac{\partial}{\partial W_{t,j}} \left[ \int_{i \in j} W_t(i)^{1-\eta} di + \int_{i \notin j} W_t(i)^{1-\eta} di \right]^{\frac{1}{1-\eta}} = \frac{1}{n} \left( \frac{W_{t,j}}{W_t} \right)^{-\eta} \tag{14}
$$

In the appendix we show that at the symmetric equilibrium, the wage-setters’ first order conditions yield:

$$
E_t \left[ \frac{1}{C_{t,i} - hC_{t-1,i}} - \frac{\beta h}{C_{t+1,i} - hC_{t,i}} \right] \left( 1 - \eta \right) + \frac{(\eta - \alpha^{-1})}{n} = -\chi L_t^\eta \left[ (\eta - \eta - \alpha^{-1}) n^{-1} \right] \left( 1 - \tau_L \right) W_t \tag{15}
$$

This implies that labor supply is

$$
W_t^* = -\nu E_t \frac{U_{Lt}}{U_{Ct}} \frac{1}{1 - \tau_L} \tag{16}
$$

where $\nu = \frac{\eta n - \gamma + \alpha^{-1}}{(\eta - 1)n - \gamma + \alpha^{-1}}$ denotes the gross wage markup. Observe that our formulation nests alternative labor market regimes, ranging from perfect competition ($n, \eta \to \infty, v = 1$) to monopolistic competition ($n \to \infty, 1 < \eta < \infty, v = \eta (\eta - 1)^{-1}$), to strategic wage setting ($1 \leq n < \infty, 1 < \eta < \infty$).

Following Blanchard and Gali (2007) and Christofoel and Linzert (2010), we assume that real wages respond sluggishly to labor market conditions in a parsimonious way. Specifically, we assume the following partial adjustment model:

$$
W_t = (W_{t-1})^\kappa (W_t^*)^{\kappa - 1} \tag{17}
$$

where $\kappa$ is an index of real rigidities. Note that equation (17) is compatible with different theoretical specifications of the labor market. Thus, it permits

\footnote{See, e.g., Gnocchi (2009) for a similar framework. See also Soskice and Iversen (1998), Lippi (2003), Zanetti (2007).
\footnote{We restrict attention to symmetric equilibria where all wage-setters claim the same real wage.}}
us to consider the effects of real wage rigidities from a general perspective, i.e.,
abstracting from their roots.\textsuperscript{16}

The labor market clearing condition comes from the labor demand (10) and
supply (17), which is subject to a partial adjustment process, unless \( \kappa = 0 \).
Because of the labor market imperfections, in the steady state the ratio between
the marginal rate of substitution (\(-U_L/U_C\)) and the marginal product of labor
\((MPL)\) will be different from one, i.e. a labor wedge \( \vartheta \) will arise:

\[
MPL = -\vartheta E_t \frac{U_L}{U_C} 
\]

where \( \vartheta \equiv \nu \frac{1 + \gamma}{1 - \gamma} \). This wedge is an increasing function of \( \eta \) and \( n^{17} \) (i.e., the
elasticity of substitution of wage-setters’ coordination) and of the tax rates (\( \tau_S \)
and \( \tau_L \)). In other words, the labor wedge reflects, on the one hand, technology,
labor market institutions and the productive structure of a country and, on the
other hand, the taxation and social security system. In our setup, an increase
in the gross wage markup or in the tax wedge raises the cost of labor (and real
wages) and, \textit{coeteris paribus}, lowers employment.\textsuperscript{18}

The parameter \( \vartheta \) is thus a measure of the (permanent) labor market imperfections,
whereas \( \kappa \) measures the (temporary) rigidities in the wage adjustment
process.

\subsection*{2.2.4 Aggregate resource constraint}

The economy-wide resource constraint is:

\[
C_t = Y_t - I_t \left[ 1 + f \left( \frac{J_t}{J_{t-1}} \right) \right] - Y_t \bar{g}
\]

where \( \bar{g} \) is a fixed fraction of income which the government spends financing the
expenditure by taxation without any recourse to debt.

\section*{2.3 The financial sector}

As mentioned above, the representation of the financial sector is borrowed from
Gertler and Karadi (2009) and Gertler and Kiyotaki (2010). Banks are owned
by households. Each period a fraction \( \sigma \) of bankers survives while a fraction
\( 1 - \sigma \) exits and is replaced.\textsuperscript{19} Each banker’s objective is then to maximize the
expected discounted present value of its future flows of net worth \( n_t \), that is:

\[
V_t = E_t \sum_{i=1}^{\infty} (1 - \sigma) \sigma^{i-1} \Lambda_{t+i} n_{t+i}
\]
Bankers can loan the sum of the bank net worth $n_t$ and deposits $d_t$ to firms or can divert a fraction $\theta$ of this sum to their family. Diverting assets can be profitable for the banker who, afterwards, would default on his debt and shut down, and correspondingly represents a loss for creditors who, at most, could reclaim the fraction $1 - \theta$ of assets. As a consequence, depositors would restrict their credit to the banks as they realize that the following incentive constraint must hold for the banks in order to prevent them from diverting funds:

$$V_t(s_t, d_t) \geq \theta (n_t + d_t) \quad (21)$$

i.e., the value of the bank must always be greater than the amount the banks can divert.

Each period, the value of loans funded, $Q_t s_t$, must equal the sum of the bank net worth $n_t$ and deposits $d_t$:

$$Q_t s_t = n_t + d_t \quad (22)$$

where $s_t$ is the volume of loans funded. Recall that the bank’s loans can be interpreted as firms’ equities owned by the bank.

The net worth for the single bank evolves according to:

$$n_t = \Psi_t[Z_t + (1 - \delta) Q_t] s_t - R_t d_{t-1} \quad (23)$$

where $Z_t$ is the dividend payment at $t$ on the loans the bank funded at time $t - 1$. It is worth noticing that $\Psi_t$ affects the value of the capital of the non-financial firms and, in turn, the value of the equities held by the bank.

The solution of the above dynamic optimization problem implies

$$Q_t s_t = \phi_t n_t \quad (24)$$

as

$$\mu_t = \frac{v_t}{Q_t} - v_t > 0 \quad (25)$$

$$\phi_t = \frac{v_t}{\theta - \mu_t} \quad (26)$$

where $\phi_t$ is the leverage ratio of the bank; $v_t$ is the marginal value of assets for the banks; and $v_t$ is the marginal value of deposits to the bank at time $t$.

As banks are constrained on the retail deposit market, there will be a positive difference between the marginal value and cost of loans for the banks. Moreover, the marginal value of net worth $\Omega_t$ and the gross rate of return on bank assets $R_{kt+1}$ must obey the following conditions:

$$v_t = E_t \Lambda_{t,t+1} \Omega_{t+1} R_{t+1} \quad (27)$$

$$\mu_t = E_t \Lambda_{t,t+1} \Omega_{t+1} (R_{kt+1} - R_{t+1}) \quad (28)$$

with

$$\Omega_{t+1} = 1 - \sigma + \sigma(v_{t+1} + \phi_{t+1} \mu_{t+1}) \quad (29)$$

$$R_{kt+1} = \Psi_{t+1} \frac{Z_{t+1} + (1 - \delta) Q_{t+1}}{Q_t} \quad (30)$$

---

20See Appendix B for details on the derivation.
21The term $\Lambda_{t,t+1} \Omega_{t+1}$ can be thought of as the augmented stochastic discount factor since it accounts for the stochastic marginal value of the net worth ($\Omega_{t+1}$).
It follows that there will always be an excess return of assets over deposits:

\[ E_t \Lambda_{t,t+1} \Omega_{t+1} R_{t+1} > E_t \Lambda_{t,t+1} \Omega_{t+1} R_{t+1} \]  

(31)

Aggregating (24) over all banks, we obtain the sector balance sheet and the demand for assets from the banks:

\[ Q_t S_t = N_t + D_t \]  

(32)

\[ Q_t S_t = \phi_t N_t \]  

(33)

The overall bank lending capacity depends on the aggregate bank capital which, in turn, may be affected by the changing value of the funded assets.

The aggregate net worth \( (N_t) \) evolves according to

\[ N_t = (\sigma + \zeta) \Psi_t [Z_t + (1 - \delta) Q_t] S_{t-1} - \sigma R_t D_{t-1} \]  

(34)

The above expression is determined by a double aggregation. We compute the aggregate net worth of new and old bankers using (23) twice and then we sum them up. In detail, we know that the new individual bankers are endowed with a fraction \( \zeta/(1 - \sigma) \) of the value of the asset intermediated by the exiting bankers (i.e., \( (1 - \sigma) [Z_t + (1 - \delta) Q_t] S_{t-1} \)) while the surviving bankers’ net worth is equal to \( \sigma [Z_t + (1 - \delta) Q_t] S_{t-1} \).

Finally, the securities markets clear when:

\[ S_t = I_t + (1 - \delta) K_t \]  

(35)

This completes the description of the set-up of the model.

\[ 22 \text{Aggregate values for financial assets are indicated by capital letters.} \]
2.4 Equilibrium

A competitive equilibrium is a set of plans \( \{C_t, L_t, K_t, Q_t, Z_t, R_{kt}, R_t, N_t, W_t, D_t, S_t, v_t, \Omega_t, \phi_t, \mu_t\} \) satisfying the following conditions derived above:

\[
1 = \beta E_t \frac{U_{Ct+1}}{U_{Ct}} R_{t+1}
\]

\[
L_t = \left( \frac{(1 + \tau_g) W_t}{A_t K_{t-1}^\alpha (1 - \alpha)} \right)^{-\frac{1}{\gamma}}
\]

\[
Z_t = \alpha A_t \left( \frac{L_t}{K_t} \right)^{1-\alpha}
\]

\[
Q_t - 1 = f \left( \frac{I_t}{I_{t-1}} \right) + \frac{I_t}{I_{t-1}} f' \left( \frac{I_t}{I_{t-1}} \right) - E_t \Lambda_{t,t+1} f' \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{I_{t+1}}{I_t} \right)^2
\]

\[
K_{t+1} = \Psi_{t+1} (I_t + K_t (1 - \delta))
\]

\[
W_t = (W_{t-1})^\kappa \left( \frac{-v_t}{U_{Lt}} \right)^{\frac{1}{1-\gamma}}
\]

\[
C_t = A_t K_{t-1}^\alpha L_t^{1-\alpha} (1 - \bar{g}) - I_t \left[ 1 + f \left( \frac{I_t}{I_{t-1}} \right) \right]
\]

\[
\phi_t = \frac{v_t}{\theta - \mu_t}
\]

\[
v_t = E_t \Lambda_{t,t+1} \Omega_{t+1} (R_{kt+1} - R_t)
\]

\[
\mu_t = E_t \Lambda_{t,t+1} \Omega_{t+1} (R_{kt+1} - R_t)
\]

\[
\Omega_{t+1} = 1 - \sigma + \sigma (v_{t+1} + \phi_{t+1} \mu_{t+1})
\]

\[
R_{kt+1} = \Psi_{t+1} \frac{Z_{t+1}^\alpha (R_{kt+1} - R_t)}{Q_t}
\]

\[
Q_t \, S_t = N_t + D_t
\]

\[
\phi_t = \frac{Q_t \, S_t}{N_t}
\]

\[
N_t = (\sigma + \zeta) \Psi_t |Z_t + (1 - \delta) \, Q_t| S_t - \sigma R_t D_t - 1 - \delta K_t
\]

\[
S_t = I_t + (1 - \delta) K_t
\]

given the exogenous process \( \{\Psi_t\} \) and the economy initial conditions for the endogenous state variables.

3 Financial shocks and labor rigidities

3.1 Calibration

We calibrate the model at a quarterly frequency. The discount factor is set at a value consistent with a real interest rate of 4% per year. We set the Frisch labor supply elasticity \((1/\varepsilon)\) to 2 and the parameter \(\chi\) of the utility function so that households devote about one third of their time to paid work in the deterministic

\[\text{Note that (42) is obtained aggregating (5) and substituting it into (19); equation derives from (10) and (17); } v_t \text{ can be obtained from (25); } U_{Ct} \text{ and } A_{t,t+1} \text{ have been already defined and } f(\cdot) \text{ will be specified in the next section.} \]
steady state, normalizing to one the total time. The habits parameter is $0.5$. The depreciation rate ($\delta$) is 0.025 and the capital share ($\alpha$) is 0.33. We assume that the adjustment cost is

$$f\left(\frac{I_t}{I_{t-1}}\right) = \frac{\gamma}{2} \left(\frac{I_t}{I_{t-1}} - 1\right)^2$$

and use a calibration in line with Altig et al. (2011).

In the labor market, as benchmark we set the intra-temporal elasticity of substitution across labor inputs to 6, corresponding to a wage markup of 20%. In the baseline case, we consider a small degree of workers' coordination in setting their actions (i.e., they behave closely to atomistic wage-setters) by setting $1/n = 0.33$. The corresponding wage markup is 1.25. We also consider a small tax wedge by setting it to 1.20. We do not report the index of real wage rigidities ($\kappa$) as different calibrations will be considered. Specifically, in the next subsection, we only focus on labor market imperfections and assume real wages adjust immediately ($\kappa = 0$). In the next subsection instead we focus on real rigidities by using the baseline calibration under different assumption about $\kappa$, ranging 0 from to 0.7.

Regarding the financial sector, we calibrate $\sigma$ to obtain an average banks survival period of ten years; $\theta$ and $\zeta$ to meet an economy-wide leverage ratio of about four and an average credit spread of one hundred basis points per year. We finally assume that in the steady state government consumption represents 20% of value added. Besides, our baseline model allows for perfectly flexible real wages; we analyze the effect of different levels of rigidities in a separate section.

The values we assign to the structural parameters in the baseline calibration of the model are summarized in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>Discount rate</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>0.5</td>
<td>Inverse Frisch labor supply elasticity</td>
</tr>
<tr>
<td>$\chi$</td>
<td>5.584</td>
<td>Relative utility weight of labor</td>
</tr>
<tr>
<td>$h$</td>
<td>0.5</td>
<td>Habits parameter</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.33</td>
<td>Effective Capital share</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.025</td>
<td>Depreciation rate</td>
</tr>
<tr>
<td>$1/\gamma$</td>
<td>0.4</td>
<td>Elasticity of investment to the price of capital</td>
</tr>
<tr>
<td>$\eta$</td>
<td>6</td>
<td>Elasticity of substitution across labor inputs</td>
</tr>
<tr>
<td>$1/n$</td>
<td>0.33</td>
<td>Union density</td>
</tr>
<tr>
<td>$1+\tau_o$</td>
<td>1.2</td>
<td>Tax wedge</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.39</td>
<td>Fraction of divertable assets</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.972</td>
<td>Survival rate of bankers</td>
</tr>
<tr>
<td>$\frac{\zeta}{1-\sigma}$</td>
<td>0.107</td>
<td>Transfer to new entering bankers</td>
</tr>
<tr>
<td>$\bar{g}$</td>
<td>0.2</td>
<td>Steady state government consumption</td>
</tr>
<tr>
<td>$\rho_{\omega}$</td>
<td>0.66</td>
<td>Persistence of the capital quality shock</td>
</tr>
</tbody>
</table>

Table 1 – Baseline parameter values

$^{24}$ As in Gertler and Kiyotaki (2010), Christiano et al. (2005) consider 0.65.

$^{25}$ Results are robust with respect to different calibrations. Elasticity of investment to the price of capital (1/$\gamma$) usually ranges between 0.1-0.6. Further simulations are available upon request.

$^{26}$ Alternative choices will be later discussed.

$^{27}$ See Gertler and Kiyotaki (2010) for a discussion.
3.2 Labor market imperfections

In order to isolate real wage rigidities from labor market imperfections in this section we set $\kappa = 0$, i.e. we assume no partial adjustment for real wages.

Equilibrium in the labor market is then given by the following equation:

$$ (1 - \alpha) A_t K_{t-1} L_t^{-\alpha} = \vartheta \chi L_t E_t \left[ 1 \frac{1}{C_t - hC_{t-1}} - \frac{\beta h}{C_{t+1} - hC_t} \right] $$

By using (52) we consider three different scenarios: the competitive wages case; our baseline calibration; an economy with larger imperfections where we set $\eta = 4.4$ (the resulting markup is 1.35). It is worth noticing that in all scenarios we consider tax distortions as the tax wedge is set according to our baseline calibration.

Figure 1 displays the impulse response functions to a negative capital quality shock in the three scenarios.

Concerning the labor market imperfections the result is clear. The dynamics of the model is independent of the degree of imperfections in the labor market.\(^{28}\)

\[^{28}\text{Recall that the markup is also influenced by the wage-setter coordination.}\]

\[^{29}\text{The figure displays per cent deviations from the steady state. Output response is computed from the production function, after aggregation.}\]

\[^{30}\text{Although the three scenarios exhibit the same dynamics, the effects at the levels are different. Specifically, the economy with more imperfections will be characterized by lower}\]
A sort of neutrality of the labor market institutions with respect to the propagation in the economy of financial disturbances arises. Specifically, in all the three cases the dynamics is the same. As in Gertler and Kiyotaki (2010), the financial shock triggers a financial accelerator\(^{31}\) that implies a fall in the investment activity as well as in the other real variables because of the reduction in the value of the net worth of banks (which entails a rise in the external finance premium) and thus in their capacity of collecting deposits. The fall in investment and the disruption of financial intermediation lead to a fall in labor demand and a consequent fall in output and consumption.

What emerges is that the effect of the financial shock is robust to changes in the degree of labor market imperfections, i.e. in the level of the labor wedge. Since the labor wedge results from the combination of various factors like the elasticity of substitution among labor kinds \((\eta)\), the degree of interaction among wage setters \((\sigma)\) and the tax rates \((\tau_S \text{ and } \tau_L)\), we can meaningfully point out that the effects of a financial shock are robust to the variations of these institutional factors and labor market features.

The *neutrality* result concerning the role played by different degrees of labor market imperfections in shaping the consequences of a financial shock in our economy may have relevant policy implications. In fact, contrary to some anecdotal evidence, according to our simulations, the institutional factors affecting the labor wedge do not alter the channels of transmission of financial frictions to the real economy.

### 3.3 Real wage rigidities

The above neutrality results is robust with respect to wage rigidities: if we set \(\kappa\) between zero and one, different degrees of labor market imperfections (either in the markup or the tax wedge) do not affect financial-shock-driven fluctuations.\(^{32}\) Different degrees of wage rigidities instead clearly lead to different dynamics of real variables, as a change in \(\kappa\) affects the dynamics, but independently of the degree of labor market imperfections.

Figure 2 describes the impulse responses triggered by a negative financial shock in our economy under different degrees of real wage rigidities. In simulations, we use our baseline calibration\(^{33}\) and consider: perfectly flexible wages \((\kappa = 0)\); an intermediate level of real wage rigidity \((\kappa = 0.4)\); and an economy with a very slow adjustment process for real wages \((\kappa = 0.7)\).

\(^{31}\)Differently from the frictionless case that is not reported. See Gertler and Kiyotaki (2010).

\(^{32}\)Further simulations are available upon request.

\(^{33}\)However, the figure is robust with respect to a change in the wage markup or in the tax wedge, because, as already said, labor market imperfections are neutral with respect to financial shock dynamics.
From a qualitative point of view, all the variables react to the shock exactly as described in the previous section, but the clear-cut result reported in Figure 2 is that, differently from other labor market imperfections, real wage rigidities amplify the effects of the financial crisis. In particular, real wage rigidities amplify investment, output and hour dynamics.

Larger real wage rigidities are associated with both deeper fluctuations and slower recoveries of the variables with respect to their steady state path.\textsuperscript{34} In fact, these rigidities interact with financial frictions by affecting the net worth of the banks, thus worsening the reaction of the economy to a negative financial shock. Our result can be intuitively explained as follows: when the quality of capital worsens, the marginal productivity of labour decreases, if the real wage is not free to fall in parallel to this, employment and, therefore, the productivity of capital fall more than when the real wage is flexible. The effect is a marked reduction in the rate of return of capital, which further worsens the banks’ balance sheets, leading to a deepening of the credit contraction.

The quantitative impact of real wage rigidities is described by the following table. The table is built by considering 200,000 simulations for each different

\textsuperscript{34}It is worth noticing that larger welfare losses are associated to larger fluctuations (see, for a discussion, Gertler and Karadi, 2009; Gertler and Kiyotaki, 2010).
degree of wage rigidity (columns) and reports the volatilities of output, consumption, investment and hours (rows). The first column reports the variance ($\sigma^2$) of the variables in the no real rigidity, NRR, case (i.e., $\kappa = 0$); other columns report variances ($\sigma^2$) and per cent differences in variances ($\Delta \sigma^2\%$) with respect to the NRR case when $\kappa = 0.4$ and $\kappa = 0.7$.

<table>
<thead>
<tr>
<th></th>
<th>NRF</th>
<th>$\kappa = 0.4$</th>
<th>$\kappa = 0.7$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma^2$</td>
<td>$\Delta \sigma^2%$</td>
<td>$\sigma^2$</td>
</tr>
<tr>
<td>Output</td>
<td>0.73</td>
<td>0.87</td>
<td>0.19</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.67</td>
<td>0.69</td>
<td>0.03</td>
</tr>
<tr>
<td>Investment</td>
<td>0.19</td>
<td>0.23</td>
<td>0.21</td>
</tr>
<tr>
<td>Hours</td>
<td>0.07</td>
<td>0.08</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Table 2 – RBC 2nd moments** and real rigidities

Table 2 shows that the economies characterized by more rigid labor markets experience higher volatilities associated to financial instability and that the differences are not negligible.

4 Conclusions

By using the recent Gertler and Kiyotaki’s (2010) setup, this paper has explored the interaction between distortions in labor market institutions and in the financial sector. We found that labor market imperfections such as workers’ monopoly power and/or strategic interactions among wage setters or fiscal institutions determining tax wedges have no impact on the volatility of the real economy induced by a financial shock. Instead, real wage rigidities matter, as they amplify the effects of financial imperfections.

Our results have serious implications for the policy debate about the role of labor markets in amplifying or dampening the current financial crisis. Amplifications of financial shocks are related only to those features of the institutional setting that affect the dynamic adjustment process of wages. Economies with larger imperfections in labour markets will not systematically observe larger or smaller recessions, unless a positive correlation between those imperfections and real wage rigidities is introduced.36

This implies that one should carefully detect the different features of labour markets in order to predict the likely effects of financial frictions on volatility.

Appendix A – Unions’ problem

Each union $j$ set the wage $W_{t,i,j}$ of the agent $i \in j$, (i.e., $W_{t,i} = W_{t,i,j}$ if $i \in j$) so as to maximize its utility in (1), subject to the budget constraint (2), and the constraints (8), (7) and (9). Using the last two equations we can write $L_{t,i} = \left( \frac{W_{t,i}}{W_t} \right)^{-\eta} \left( \frac{1+\sigma}{\lambda_t K_t}\right)^{1-\alpha} W_t^{\frac{1}{2}}$. Substituting this expression for $L_{t,i}$ in 1 and (2), we see that choosing $W_{t,j}$ so as to equalize in expectation the

$${}^{35}$$Note that variances reported are multiplied by 100.

$${}^{36}$$However, this does not seem empirically to be the case. For instance, Belgium, Germany and Italy are characterized by high degrees of union density and coverage as well of tax wedges, but Germany and Italy, in contrast to Belgium, have a very low wage indexation, which is one of the factors influencing real wage flexibility (see Du Caju et al., 2008).
marginal cost and the marginal benefit of working implies maximizing w.r.t. \( W_{t,j} \) the following expected value:

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left[ -\frac{\chi}{1+\varepsilon} W_t^{-\eta(1+\varepsilon)} W_t^{\left(\frac{\eta - \frac{1}{\alpha}}{1+\varepsilon}\right)(1+\varepsilon)} (1+\tau_s)^{-\frac{1+\varepsilon}{\alpha}} \left[ A_t K_{t\rightarrow t-1}^\alpha (1-\alpha) \right]_{\frac{1+\varepsilon}{\alpha}}^t + \right. \\
\left. U_{Ct}(1+\tau_s) \left( 1 - \tau_t \right) W_t^\eta W_t^{1-\eta} (A_t K_{t\rightarrow t-1}^\alpha (1-\alpha) \right]_{\frac{1}{\alpha}}^t \right]
\]

given (8). Equating to zero the derivative of this expected value w.r.t. \( W_{t,j} \); using (14) we find:

\[
-\chi \left[ A_t K_{t\rightarrow t-1}^\alpha (1-\alpha) \right]_{\frac{1+\varepsilon}{\alpha}}^t \left[ -\eta W_t^{-\eta(1+\varepsilon)-1} W_t^{\left(\frac{\eta - \frac{1}{\alpha}}{1+\varepsilon}\right)(1+\varepsilon)} + \right. \\
\left. \left( \eta - \frac{1}{\alpha} \right) W_t^{-\eta(1+\varepsilon)} W_t^{\left(\frac{\eta - \frac{1}{\alpha}}{1+\varepsilon}\right)(1+\varepsilon)-1} \frac{W_t}{W_t} \right) - \eta \right] + \\
\left. \frac{U_{Ct}(1-\tau_t) W_t^\eta W_t^{1-\eta} (A_t K_{t\rightarrow t-1}^\alpha (1-\alpha) \right]_{\frac{1}{\alpha}}^t + \right. \\
\left. \left( \eta - \frac{1}{\alpha} \right) W_t^{1-\eta} W_t^{\eta-1/\alpha-1} \frac{W_t}{W_t} \right]_{\frac{1}{\alpha}}^t = 0
\]

In a symmetric equilibrium \( \frac{W_t}{W_{t,j}} = 1 \), so after some simplifying we can write this as:

\[
\chi \left[ A_t K_{t\rightarrow t-1}^\alpha (1-\alpha) \right]_{\frac{1}{\alpha}}^t \left[ \eta W_t^{-\frac{\eta}{2} - 1} - \frac{\eta - \alpha^{-1}}{n} W_t^{-\frac{\eta}{2} - 1} \right] + \\
\left. U_{Ct}(1-\tau_t) \left[ (1-\eta) + \frac{\eta - \alpha^{-1}}{n} \right] = 0
\]

or using again (9) to eliminate \( A_t K_{t\rightarrow t-1}^\alpha (1-\alpha) = (1+\tau_s) W_t^\eta L_t^\alpha \) and simplifying:

\[
\chi W_t^{-\frac{\eta}{2}} L_t^\alpha \left( \eta - \frac{\eta - \alpha^{-1}}{n} \right) + (1-\tau_t) U_{Ct} \left( (1-\eta) + \frac{\eta - \alpha^{-1}}{n} \right) = 0
\]

This, recalling (4), gives us (15) in the text.

**Appendix B – Financial sector appendix**

**B1 – Banker’s maximization problem**

The objective of the bank at the end of period \( t-1 \) is the expected present value of future dividends, as follows:

\[
V_{t-1}(s_{t-1}, d_{t-1}) = E_{t-1} \sum_{i=1}^{\infty} (1-\sigma) \sigma^{i-1} A_{t\rightarrow t-1, i} s_{t-1, i} d_{t-1, i}
\]
Given the (sequence of) balance sheets constraints:

\[ Q_{t-1}s_{t-1} - n_{t-1} = d_{t-1}. \]  

(54)

we can formulate the following Bellman equation:

\[ V_{t-1}(s_{t-1},d_{t-1}) = \mathbb{E}_{t-1} \Lambda_{t-1,t} \{(1 - \sigma)n_t + \sigma[\max_{s_t,d_t} V_t(s_t,d_t)]\} \]

(55)

The net worth at \( t \), \( n_t \), i.e. the gross payoff from assets funded at \( t-1 \), net of borrowing costs, evolves according to

\[ n_t = \psi_t[Z_t + (1 - \delta) Q_t]s_{t-1} - R_t d_{t-1}. \]

(56)

By combining (54) and (56), we can then write:

\[ Q_t s_t - d_t = d_{t-1} = \psi_t[Z_t + (1 - \delta) Q_t]s_{t-1} - R_t d_{t-1}. \]

(57)

Given the incentive constraint stemming from the agency problem:

\[ V_{t-1}(s_{t-1},d_{t-1}) > \theta Q_{t-1}s_{t-1}, \]

(58)

to solve the maximization problem of the banker we define the Lagrangian \( L \):

\[ L = \mathbb{E}_{t-1} \Lambda_{t-1,t} \{(1 - \sigma)\psi_t[Z_t + (1 - \delta) Q_t]s_{t-1} - R_t d_{t-1} + \sigma[V_t(s_t,d_t) + \lambda_t[V_t(s_t,d_t) - \theta Q_t s_t]]\}. \]

(59)

This has to be maximized given the constraint (57). To do so we formulate the following guess for the value function:

\[ V_t(s_t,d_t) = v_{st}s_t - v_t d_t. \]

(60)

The derivative of (59) with respect to \( d_t \) (of which \( s_t \) is a function, given (57)) must equal zero, for an interior solution. This gives us, by using (57) to calculate the derivative of \( s_t \) with respect to \( d_t \), the following condition:

\[ \mathbb{E}_{t-1} \Lambda_{t-1,t} \left(-\lambda_t \theta - \frac{\partial V_t(s_t,d_t)}{\partial d_t} (1 + \lambda_t)\right) = 0 \]

(61)

or, assuming (60):

\[ -\theta \lambda_t + v_t (1 + \lambda_t) = 0 \]

(62)

The constraint (58) can be written, using (60), as \( v_{st}s_t - v_t d_t \geq \theta Q_t s_t \) and, by (by using (54)):

\[ v_t n_t \geq Q_t s_t \left(\theta + v_t - \frac{v_{st}}{Q_t}\right) \]

(63)

so assuming this constraint holds as an equality we deduce: \( V_t(s_t,d_t) = v_{st}s_t - v_t (Q_t s_t - n_t) = \frac{(v_{st} - v_t Q_t) n_t}{Q_t(\theta + v_t - \frac{v_{st}}{Q_t})} + v_t n_t \). Hence:

\[ V_t(s_t,d_t) = v_t n_t \left(\frac{\mu_t}{\theta - \mu_t} + 1\right) \]

(64)

where \( \mu_t = \frac{v_{st}}{Q_t} - v_t > 0 \).
If we define:

$$\phi_t = \frac{v_t}{\theta - \mu_t}$$  \hspace{1cm} (65)

it follows that

$$V_t(s_t, d_t) = n_t (\mu_t \phi_t + v_t)$$  \hspace{1cm} (66)

By substituting the above expression (66) for $V_t(s_t, d_t)$ in (55), we have:

$$V_t(s_t, d_t) = E_t \Lambda_{t+1, t} \left[ (1 - \sigma)n_{t+1} + \sigma n_t (\mu_{t+1} \phi_{t+1} + v_{t+1}) \right]$$

or

$$V_t(s_t, d_t) = E_t \Lambda_{t, t+1} \Omega_{t+1} n_{t+1}$$  \hspace{1cm} (67)

where

$$\Omega_{t+1} = (1 - \sigma) + \sigma (\mu_{t+1} \phi_{t+1} + v_{t+1})$$  \hspace{1cm} (68)

and using (56):

$$V_t(s_t, d_t) = E_t \Lambda_{t, t+1} \Omega_{t+1} \left( \psi_{t+1} \left[ Z_{t+1} + (1 - \delta) Q_{t+1} s_t - R_{t+1} d_t \right] \right)$$  \hspace{1cm} (69)

So by the method of undetermined coefficients it follows that:

$$v_t = E_t \Lambda_{t, t+1} \Omega_{t+1} R_{t+1}$$  \hspace{1cm} (70)

and

$$v_{st} = E_t \Lambda_{t, t+1} \Omega_{t+1} \left\{ \psi_{t+1} \left[ Z_{t+1} + (1 - \delta) Q_{t+1} \right] \right\}.$$  \hspace{1cm} (71)

**B2 – Assets demand**

We can rewrite (63), given (25), as:

$$(\theta - \mu_s) Q_t s_t = v_n n_t$$  \hspace{1cm} (72)

The individual bank total demand for assets $Q_t s_t$ can then be written, using (25) as:

$$Q_t s_t = \phi_n n_t$$  \hspace{1cm} (73)

which, at the aggregate level, turns out to be:

$$Q_t S_t = \phi_N N_t$$  \hspace{1cm} (74)
Appendix C – Steady state

In the steady state the model is defined by the following two blocks of equations. Concerning the real part of the economy, from equations (36)-(42), we have:

\[ R = \frac{1}{\beta} \] (75)
\[ L = \left( \frac{1 + \tau_S}{AK^\alpha (1 - \alpha)} W \right)^{-\frac{1}{\pi}} \] (76)
\[ Z = \alpha A \left( \frac{L}{K} \right)^{1-\alpha} \] (77)
\[ Q = 1 \] (78)
\[ I = \delta K \] (79)
\[ W = \frac{\nu \chi L^\varepsilon (1 - h) C}{1 - \tau_L - 1 - \beta h} \] (80)
\[ C = \left[ (1 - \bar{g}) A \left( \frac{L}{K} \right)^{1-\alpha} - \delta \right] K \] (81)

Concerning the financial sector of the economy, from equations (43)-(51), we have:

\[ \phi = \frac{v}{\theta - \mu} \] (82)
\[ v = \Omega \] (83)
\[ \mu = \Omega (\beta R_k - 1) \] (84)
\[ \Omega = 1 - \sigma + \sigma (v + \phi \mu) \] (85)
\[ R_k = Z + (1 - \delta) \] (86)
\[ S = N + D \] (87)
\[ \phi = S/N \] (88)
\[ N = (\sigma + \zeta) [Z + (1 - \delta)] S - (\sigma / \beta) D \] (89)
\[ S = K \] (90)

Some cumbersome algebra is then requested to obtain the steady state. By using equations (84), (83), (82), one obtains:

\[ \phi = \frac{\Omega}{\theta - \Omega (\beta R_k - 1)} \] (91)

which substituted in (85) yields:

\[ \Omega = 1 - \sigma + \frac{\sigma \Omega \theta}{\theta - \Omega (\beta R_k - 1)} \] (92)

Combining (75), (86), (87) and (89) gives:

\[ N = \frac{(\sigma + \zeta) R_k - (\sigma / \beta)}{1 - (\sigma + \zeta) R_k} D \]
which solved for $D$ and substituted in (87), given (90), after rearranging yields:

$$K = \left[ 1 + \frac{1 - (\sigma + \zeta) R_k}{(\sigma + \zeta) R_k - \sigma/\beta} \right] N$$  \hspace{1cm} (93)

that combined with (88) and (90) yields:

$$\phi = 1 + \frac{1 - (\sigma + \zeta) R_k}{(\sigma + \zeta) R_k - \sigma/\beta}$$  \hspace{1cm} (94)

By combining (91) and (94), we get the following expression for $R_k$:

$$R_k = \frac{\beta \Omega + \theta (\beta - \sigma)}{(\zeta + \beta) \Omega \beta}$$  \hspace{1cm} (95)

Equations (92) and (95) are a two equation system in two unknowns, $\Omega$ and $R_k$. The solution of the system clearly gives the steady state values for these two variables.

By substituting (95) into (92), one obtains the following second order polynomial equation in $\Omega$:

$$\zeta \Omega^2 + [\zeta(\theta - 1)(1 - \sigma)\theta \sigma(1 - \beta)]\Omega - (1 - \sigma)(\zeta + \sigma) \theta = 0$$  \hspace{1cm} (96)

whose positive solution is chosen and substituted in (95) to obtain $R_k$. Once system (92) and (95) is solved, the steady-state values for $\mu$, $\phi$, and $\nu$ are obtained straightforwardly.

Finally, by combining (80), (76), (81), and using (77) and (86), after cumbersome algebra, we get the expression for $L$ only in terms of $R_k$:

$$L = \frac{(1 - \tau_L)(1 - \alpha)\left(\frac{R_k - \delta}{\alpha} \frac{1 - \delta}{1 - \gamma}\right)^{1/\gamma}}{\nu (1 + \tau_S) \chi \left[(1 - \bar{g}) R_k \frac{1 + \delta}{\alpha} - \delta\right]}$$  \hspace{1cm} (97)

Combining (77) and (86), and using the steady-state values for $L$ and $R_k$, $K$ is also obtained. Other steady-state values ($S$, $I$, $C$, $W$, $D$, $N$, $Z$) are then easily found recursively.

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