Theory and practice of contagion in monetary unions. Domino effects in EU Mediterranean countries: The case of Greece, Italy and Spain

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Theory and Practice of Contagion in Monetary Unions. Domino Effects in EU Mediterranean Countries: the Case of Greece, Italy and Spain

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Abstract

This paper analyzes strategic interactions and contagion effects in countries joined to a monetary union. Using game theory and a cost-benefit analysis, the paper determines the set of equilibrium solutions under which country-specific shocks are transmitted to other member countries giving rise to contagion. Numerical simulations, obtained by a simple calibration of the model on some key Mediterranean countries of the Euro Zone, show the probabilities of contagion from Greece to Spain and Italy.

JEL codes: F30, F31, F41, G01.

Keywords: Shadow exchange rate, currency crisis, monetary unions, contagion, Nash equilibria.

1 Introduction

The spectacular increase of sovereign risk premium in the Euro Area following the Greek debt explosion in 2009 and its impact on the sustainability of the European Monetary Union (EMU) have kindled research interest on
contagion effects and risk of a systemic crisis in the Euro Zone. The issue has also been of a major concern to European policymakers, and it has brought the containment of systemic financial crises to the center of an international policy debate.

There is now a substantial literature dealing with the European debt crisis, the bulk of it focusing on the role played by fiscal and financial variables in the outbreak of crisis.\footnote{See, e.g., Attinasi \textit{et al}. (2009), Barrios \textit{et al}. (2009), Haugh, Ollivaud and Turner (2009), Aizenman \textit{et al}. (2011), Amisano and Tristani (2011), Ejsing \textit{et al}. (2011), Bernoth \textit{et al}. (2012), Borgy \textit{et al}. (2012).} There is also a number of recent papers focusing on the role spillover and contagion effects played in exacerbating the sovereign debt problems in the Euro Zone.\footnote{See, e.g., Caceres \textit{et al}. (2010), Arezki, Candelon and Sy (2011), Favero and Missale (2011), Hui and Chung (2011), Arghyrou and Kontonikas (2012), Constancio (2012), De Grauwe and Ji (2012), De Sanctis (2012), Gómez-Puig and Sosvilla-Rivero (2012), Metiu (2012).} A major deficiency of this literature is that it still lacks a rigorous theoretical framework to understand the full set of events characterizing the sovereign debt crisis and contagion effects within the EMU. Exceptions are found in Bolton and Jeanne (2011), who propose a model of contagious sovereign debt crises in financially integrated economies. Arghyrou and Kontonikas (2012), who provide a model of Euro Zone crisis that combines the features of both second- and third-generation approaches to currency crisis. De Grauwe and Ji (2012), who present a model of EMU sovereign crisis inspired by the Obstfeld (1994, 1997) model of self-fulfilling speculative attacks. Canofari \textit{et al}. (2012a), who propose a model of crisis and contagion within monetary unions that combines the features of both first- and second-generation approaches to currency crisis.\footnote{A detailed analytical discussion of the basic theoretical approaches to currency and financial crises can be found in Piersanti (2012).}

This papers adds to this strand of literature by providing a new theoretical framework to understand crises and contagion phenomena within
monetary unions. Specifically, we apply game theory and a cost-benefit analysis to study voluntary exits and contagion effects in countries joined to a monetary union. The paper looks at two non-core, or periphery countries of a large union and examines the role of strategic interactions and structural asymmetries in determining the set of equilibrium solutions following a country-specific shock.

The paper is organized as follows. Section 2 discusses the theoretical framework. Sections 3 applies the model to the case of Euro Mediterranean countries to assess the probability of contagion under different assumptions about the exit costs. Section 4 concludes.

2 Theoretical framework

To analyze voluntary exits and contagion effects in a monetary union, we look at a stochastic economy and a currency-union game where, conditional on the realization of exogenous shocks that cause deviations of the target variables from their preferred level, each member country can either choose to remain in the monetary union or exit by comparing the welfare losses arising from alternative policy regimes.

Specifically, we consider a monetary union where two periphery members countries (A and B) strategically interact. Consistent with the second-generation approach, we also include a costly escape clause in the policymaker’s loss function. The social loss is given by a standard function defined over the output gap and inflation (measured by the change in nominal exchange rate).

Formally, measuring all variables in logs, letting the nominal exchange rate be defined as the price of the foreign currency in terms of the local currency and assuming that prices are fixed for simplicity, we can summarize the model structure by the following equations.\footnote{A more detailed discussion is in Canofari \textit{et al.} (2012b). Similar theoretical structures}
The policymaker’s loss function in country \( i \) \( (L^i_t) \) is

\[
L^i_t = (y^i_t - \bar{y}^i)^2 + \theta^i (s^i_t - \bar{s}^i)^2 + \delta C^i, \quad i \in \{A, B\},
\]

(1)

where \( y^i_t \) is date \( t \) output and \( \bar{y}^i \) the policy maker’s output target, taken to be the natural output level; \( s^i_t \) is the nominal exchange rate for country \( i \) and \( \bar{s}^i \) the rate at which the country has joined the monetary union; \( \theta^i \) is the inflation aversion coefficient; \( C^i \) is the opting out cost, assumed to be fixed for simplicity, and \( \delta \) a dummy variable defined as \( \delta = 0 \) if country \( i \) remains in the monetary union, so that \( s^i_t = \bar{s}^i \), and \( \delta = 1 \) if it exits and \( s^i_t \neq \bar{s}^i \).

The aggregate supply for country \( i \) is given by

\[
y^i_t = a(\Delta s^i_t - E_{t-1} \Delta s^i_t) + \bar{y}^i, \quad i \in \{A, B\},
\]

(2)

letting output be a function of the real wage and nominal rigidities be present in the form of a one period wage contract, where \( \Delta s^i_t \) and \( E_{t-1} \Delta s^i_t \) denote the actual and the expected change in the exchange rate for country \( i \) at time \( t \), respectively.

The international demand for the goods produced in country \( i \), \( d^i_t \), is described by

\[
d^i_t = \sigma_i \left[ s^i_t - \beta_j s^j_t - (1 - \beta_j - \gamma_i) s^W \right] - u^i_t, \quad i, j \in \{A, B\}, \quad i \neq j,
\]

(3)

where \( [s^i_t - \beta_j s^j_t - (1 - \beta_j - \gamma_i) s^W] \) is the real exchange rate, defined as a trade-weighted variable with weight \( \beta_i \) for the other small periphery country, \( \gamma_i \) for the monetary union, and \( 1 - \beta_i - \gamma_i \) for the the rest of the world, \( s^W \) is the world exchange rate, assumed fixed for simplicity, and \( u^i_t \) is an

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\( ^5 \)The cost \( C \) may have several sources and could reflect, for example, the loss of international reputation and anti-inflation credibility, voter dissension, or removal from the office.

are also found in Masson (1999), Buitet et al. (2001), Berger and Wagner (2005), Canofari et al. (2012a).
i.i.d. random shock described by a continuous, bell-shaped and symmetric (around zero) probability density function.

Goods market equilibrium in each country leads to

\[ y_i = \sigma_i \left( s_i^t - \beta_i s_i^t - (1 - \beta_i - \gamma_i)s^W \right) - u_i^t, \quad i, j \in \{A, B\} \quad i \neq j, \quad (4) \]

which can be rewritten as

\[ y_i^F = \sigma_i \left(s_i^t - s_i^t\right) + y_i^{i,F} \]
\[ y_i^{i,F} = \sigma_i \left[ s_i^i - \beta_i s_i^j - (1 - \beta_i - \gamma_i)s^W \right] - u_i^i, \quad i, j \in \{A, B\} \quad i \neq j \]

when the two countries join the currency union at a nominal fixed parity \( \bar{s}_i \),
where \( y_i^{i,F} \) denotes the output for country \( i \) required to stay in the 'hard' monetary arrangement. The incentive to exit for each country thus comes from the increase in output which can be obtained by a change of the exchange rate.

Notice that, under \( u_t = 0 \), the exchange rate parities consistent with a monetary union agreement are endogenously determined in our model. Letting, to simplify algebra, \( \sigma_i = \sigma_j = \sigma, \bar{y}^i = \bar{y}^j = \bar{y} \), and normalizing the world exchange rate level to one, the required parities can be computed as \( \bar{s}_i^t = \frac{1}{\sqrt{n}} \bar{y} , \quad i \in \{A, B\} \). Under these rates, if no shock occurs at time \( t \), the model implies that the policymakers will have no incentive to inflate the economy to drive output above the private sector preferred level as the output targets coincide. Furthermore, as \( y_i^t = y_i^{i,F} = y_i^F = y_i^{i,F} = \bar{y} \) in this case, they will have no incentive to exit and divert from the agreed fixed parities.\(^6\)

To keep things simple we now normalize the output target levels, the world exchange rate and the nominal fixed parities to one\(^7\) and focus on a currency-union game where, following a country-specific shock, each country

\(^6\)It is easy to check that, if \( u_t^i = u^j_t = 0 \), a rational expectation equilibrium implies \( \Delta s_i^t = \Delta s_i^t = E_{t-1} \Delta s_i^t = E_{t-1} \Delta s_i^t = 0 \) and \( y_i^t = y_i^{i,F} = y_i^F = y_i^{i,F} = \bar{y} \).

\(^7\)Thus, \( y^A = y^B = 0 \), and \( s^W = \bar{s}^A = \bar{s}^B = 0 \).
can choose between two actions: either choosing to remain in the monetary union and set $\Delta s = 0$, denoted by *No exit*, or choosing to leave it and set $\Delta s \neq 0$, denoted by *Exit*. Therefore, the key issue for the authorities is to find the value of the shock at which it is optimal to operate a regime shift. However, as policy decisions are not independent in this setting, an interaction between the policymakers optimizing behavior necessarily develops, and this can lead to multiple equilibria and contagion effects across countries.

The solutions to this game arise from strategy profiles that form a Nash equilibrium. This requires solving first the model to obtain the optimal policies and corresponding losses for each country given the strategy followed by the other country, and then finding the incentive to deviate from it to check if it is or not a Nash equilibrium.

The structure of game allow us to identify four regimes: no countries exit (regime $N$); only country A exits (regime $D^A$); only country B exits (regime $D^B$); both countries exit (regime $E$). Losses tied to the foregoing scenarios are shown in the following matrix

<table>
<thead>
<tr>
<th>A / B</th>
<th>No exit</th>
<th>Exit</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Exit</td>
<td>$L^A_N$, $L^B_N$</td>
<td>$L^A_D$, $L^B_D$</td>
</tr>
<tr>
<td>Exit</td>
<td>$L^A_{D^A}$, $L^B_{D^A}$</td>
<td>$L^A_E$, $L^B_E$</td>
</tr>
</tbody>
</table>

where $L^i_j$ denotes the loss for policymaker $i$ in regime $j$, and it can be used to find the unique or multiple Nash equilibrium solutions of the game. These equilibria are stated formally in the following

**Proposition 1** Regime $E$ is a Nash equilibrium if and only if $L^A_{D^A} - L^A_E > 0$ and $L^B_{D^A} - L^B_E > 0$. Regime $N$ is a Nash equilibrium if and only if $L^A_{D^A} - L^A_N > 0$ and $L^B_{D^A} - L^B_N > 0$. Regime $D^B$ is a Nash equilibrium if and only if $L^A_E - L^A_{D^B} > 0$ and $L^B_N - L^B_{D^B} > 0$. Regime $D^A$ is a Nash equilibrium if and only if $L^A_N - L^A_{D^A} > 0$ and $L^B_E - L^B_{D^A} > 0$.$^8$

$^8$The proof is straightforward.
The alternative losses are computed solving the policymakers’ optimization problem in each regime. Under the Exit option, optimal policy for country \( i \in \{A, B\} \) is obtained by minimizing (1) with respect to \( s_i \) subject to (4) under \( \delta = 1 \); otherwise \( s_i - \bar{s} = \delta = 0 \) under the No Exit choice. The associated reaction functions are:

\[
\begin{aligned}
    s_i^e &= \frac{(u_i^* + \sigma_i^* s_i^* \sigma_i^*)}{\sigma_i^* + \theta_i} \quad \text{for} \quad \delta = 1, \\
    s_i^e &= 0 \quad \text{otherwise}
\end{aligned}
\]

Using (6), (4) and (1), the respective losses for country \( i \in \{A, B\} \) in all the regimes are:

\[
\begin{align*}
    L_N^i &= (u_i^*)^2 \\
    L_D^i &= \frac{\theta_i}{\sigma_i^* + \theta_i} (u_i^*)^2 + C^i, \quad i \in \{A, B\} \\
    L_D^{ij} &= \left( \frac{\beta_i \sigma_i^* \sigma_j^* u_j^*}{\sigma_i^* + \theta_i} + u_i^* \right)^2 \\
    L_E^i &= \frac{\theta_i (\sigma_i^2 + \theta_i)}{\theta_j (\sigma_j^2 + \theta_j) + [(1 - \beta_i^* \beta_j^*) \sigma_i^2 + \theta_i \sigma_j^2]} + C^i, \quad i \neq j \in \{A, B\}.
\end{align*}
\]

To preserve simplicity and provide a closed-form solution of the model, we now restrict the attention to the case of symmetry between countries.\(^9\)

Thus, we set \( \sigma_A = \sigma_B = \sigma, \beta_A = \beta_B = \beta, \theta_A = \theta_B = \theta \) and \( C_A = C_B = C \).

With no loss of generality, we also let \( u_i^B = 0, u_i^A = v_i \), meaning that the shock impacts on country \( A \).

With these assumptions, solving for \( v_i \) using (7)-(10), we find that the threshold values of the shock at which the government is indifferent between

\[\text{Here, } L_N^i \text{ is the loss of country } i \text{ when both countries decides to stay in, } L_D^i, \text{ is the loss of country } i \text{ when it choices to exit, } L_D^{ij}, \text{ is the loss of country } i \text{ when country } j \text{ decides to exit, and } L_E^i, \text{ is the loss of country } i \text{ when both countries opt out.} \]

\[\text{The next section will relax this assumption and extend the model to an heterogenous monetary union. Numerical simulations computing the probability of contagion across the Mediterranean countries of the EMU will also be shown.}\]
opting out and stay in the monetary union are

\[ v^* = \frac{1}{\sigma} \sqrt{(\sigma^2 + \theta)C} \]

\[ v^{**} = \frac{1}{\sigma Q} \sqrt{(\sigma^2 + \theta)C}, \]

where \( Q = \sqrt{\frac{(\sigma^2 + \theta)C}{\left[ (\sigma^2 + \theta)^2 - \sigma^4 \beta \right] - \theta (\sigma^2 + \theta)^3}} > 1, \) implying \( v^{**} > v^*. \)

We can state the main implications of the model under a country-specific shocks in the following

**Proposition 2 (Exit and contagion)** Country A exits the monetary union and devalues if and only if \( v_t > v^*; \) both policymakers exit and devalue (contagion) if \( v_t > v^{**}. \)

**Proof.** See Appendix.

The interpretation underlying this proposition is straightforward. If the shock is small enough, the stability of monetary union is preserved. A medium-size shock provides incentive to devalue only for the affected country. If the shock is large enough to be above a threshold value, contagion is observed. The proposition also suggests that both multiple equilibria and the perverse case where only the country not hit by the shock chooses to exits cannot occur.

### 3 Domino effects in Euro Mediterranean countries

In this section we focus on contagion risks in EU Mediterranean countries. We look at an heterogenous-country version of our model and examine the domino effects of an idiosyncratic shock hitting Greece (G), to estimate the probability of contagion to Spain (S) and Italy (I).

\[ \text{Note that } Q > 1 \text{ as long as } \beta \text{ is smaller than one. The first term under the root is larger than one; the second term is always larger than one for admissible values of parameters.} \]
In order to perform numerical simulations, we make use of a simple parameterization of our model under different assumptions about the exist costs. Specifically, we relied on estimates provided by Canofari et al. (2012a) for Euro Zone countries over the period 1980Q1-2010Q3 for $\sigma$, the elasticity of aggregate demand to real exchange rate, and on estimates obtained by Krause and Méndez (2008) from a panel of 34 countries over a period of 24 years for $\theta$, the inflation aversion coefficient.\footnote{Average inflation aversion coefficient over the period 1981-1997 were considered.} Moreover, we computed the weight coefficients for trade, $\beta$, as
\begin{equation}
\beta^j_i = \frac{\Pi^j_i}{\Pi^EU_i}, \quad i, j = \{G, I, S\},
\end{equation}
where $\Pi^j_i$ measures the volume of international trade (sum of exports and imports) of country $i$ with respect to country $j$, and $\Pi^EU_i$ the volume of international trade of country $i$ in the Euro Area.

The parametrization for the baseline calibration is summarized in Table 1.

| Country $i$ $\sigma_i$ $\theta_i$ $\beta^G_i$ $\beta^G_G$ |
|----------------|----------|--------|--------|
| Greece ($G$)  4.97  2.10  --  --  |
| Italy ($I$)    1.22  3.77  0.22  0.02 |
| Spain ($S$)    2.87  3.34  0.07  0.01 |

Model’s simulations were also based on the following hypotheses. We let the random shock in Greece to be normally distributed with zero mean and standard deviation equal to that of the Greek output gap over the period 2000-11.\footnote{Greek potential output was obtained by applying the Hodrick-Prescott filter to Greek GDP.} We included different values for the opting out costs. Specifically, we first computed the lowest and the highest cost by fixing the probability
of contagion and domino effects exogenously at 35% and 5%, respectively.\footnote{Calculations involved the following two equations: $\int_{v^*}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{v^*}{2\sigma^2}} du$ and $v^{**} = \frac{1}{2} Q(\sigma^2 + \theta)^C$, where $Q$ denotes the variance of the Greek output gap. Notice that the above equation for $v^{**}$ refers to the symmetric case. The expression for the asymmetric case is too muddy to be displayed here. The required computations and mathematical expressions are however available upon request.}

We then added three intermediate cost value ($c$) from a convex combination of the above maximum ($c_{max}$) and minimum cost ($c_{min}$), namely,

$$c = \chi c_{max} + (1 - \chi)c_{min}, \quad \chi \in [0.75, 0.50, 0.25].$$

The ensuing five scenarios – from the highest (1) to the lowest (5) cost – are shown in Table 2.

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>Cost cases</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>No contagion\footnote{Only Greece exits.}</td>
<td>30.57%</td>
</tr>
<tr>
<td>Spain contagion</td>
<td>14.27%</td>
</tr>
<tr>
<td>Italy and Spain contagation</td>
<td>5.00%</td>
</tr>
</tbody>
</table>

The results reveal that the probability of contagion from Greece to Spain is higher than the probability of contagion to Italy. Hence, if Italy exists due to contagion from Greece, Spain will also exits, but the reverse does not necessarily hold. It also shows that the highest probability of a full domino effect (35%) attains at the lowest opting out cost.

### 4 Conclusions

In this paper we have used game theory and a cost-benefit analysis to model contagion and domino effects in a simple monetary union.
effect of a country-specific shock, the model found that: i) small shocks do
not affect the stability of monetary union; ii) medium-size shocks provide
incentive to devalue only for the country hit by the shock; iii) contagion and
domino effects can occur only when shocks are large enough and above a
threshold level.

Numerical simulations, obtained by a simple model calibration on EMU
Mediterranean countries, provide the empirical support to the above theo-
retical implications.

Appendix – Proposition proof

Proof. Write the incentive to move between alternative regimes in a com-
 pact form as: \(a_1 = L^A_{Du} - L^A_E\); \(a_2 = L^A_{Du} - L^A_N\); \(b_1 = L^B_{Du} - L^B_E\); \(b_2 = L^B_{Du} - L^B_N\). Thus, if \(a_2 > 0\), for example, country A has no incentive to leave
the monetary union. Setting \(u_t^B = 0\), it is easy to check that

\[
\begin{align*}
  a_1 &> 0 \iff v_t > v^{***} \quad (12) \\
  b_1 &> 0 \iff v_t > v^{**} \quad (13) \\
  a_2 &> 0 \iff v_t < v^* \quad (14) \\
  b_2 &> 0 \text{ always,} \quad (15)
\end{align*}
\]

where \(v^* = \frac{1}{2} \sqrt{(\sigma^2 + \theta) C}\), \(v^{**} = Q v^*\), \(v^{***} = \frac{\beta}{\sigma^2 + \theta} v^{**}\). As \(Q > 1\), \(v^* < v^{**} < v^{***}\). Accordingly, from proposition 1 and (12)-(15), it follows that:
i) \(E\) is a Nash equilibrium if and only if \(a_1\) and \(b_1\) are both positive, i.e.
\(v_t > \max(v^{**}, v^{***}) = v^{**}\); ii) Regime \(N\) is a Nash equilibrium if and only if
\(a_2 > 0\) and \(b_2 > 0\), i.e. \(v_t < v^*\); iii) regime \(D^B\) is never a Nash equilibrium
as it would require \(b_2 < 0\).

References

of European Sovereign Debt Defaults? Fiscal Space, CDS Spreads and


