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A note on the efficacy of monetary policy under limited participation in asset market – Technical appendix

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1. The demand side

We consider a standard New Keynesian dynamic stochastic general equilibrium model augmented by rule-of-thumb consumers *a lá* Galí *et al*. (2004). We assume a continuum of infinitely-lived heterogeneous agents normalized to one. A fraction $1-\lambda$ of them consumes and accumulates wealth as in the standard setup (*savers*). The remaining fraction λ is composed by agents who do not own any asset, cannot smooth consumption, and therefore, consume all their current disposable income (*spenders*).

Formally, representative consumers are indexed by R (savers) and N (spenders), they maximize the following utility functions:

$$(1) \qquad E_t \sum_{i=0}^{\infty} \beta^i u\left(C_{t+i}^j, N_{t+i}^j\right)$$

where $\beta \in (0,1)$ is the discount factor, C_t and N_t are respectively household consumption and labor at time t. We assume the following logarithmic instantaneous utilities, $u(.) = \ln C_{t+i}^j + \kappa \ln\left(1-N_{t+i}^j\right)$ with $\kappa > 0$ which indicates how leisure is valued relative to consumption. By solving their optimization problems, consumers face the budget constraints:

$$(2) \qquad C_t^j = \frac{W_t}{P_t} N_t^j + \phi^j \left[\Pi_t^j - \frac{B_t^j - (1+i_{t-1}) B_{t-1}^j}{P_t} \right], \quad j \in \{R, N\},$$

where W_t is the nominal wage at time t and Π_t is profit sharing. ϕ^j is a binary variable such that when $j = R$, $\phi^R = 1$ and when $j = N$, $\phi^N = 0$. Note that real wages are the only source of fluctuations of spenders' disposable income and therefore they are subject to a static budget constraint, while savers are the only ones facing a dynamic constraint.[1] By solving the representative saver's and spender's maximization problem, we obtain the following first-order conditions:

(3) $$C_t^R = \left[\beta\left(1+i_t\right)P_t\right]^{-1} E_t\left[P_{t+1}C_{t+1}^R\right]$$

(4) $$C_t^N = \frac{W_t}{P_t} N_t^N$$

(5) $$W_t P_t^{-1} = \kappa C_t^j \left(1-N_t^j\right)^{-1} \qquad j \in \left\{R, N\right\}$$

Equations (3) and (4) are the optimal consumption for savers (i.e. inter-temporal stochastic consumption Euler equation) and spenders (who consume the whole labor income). Equation (5) is the optimal condition for the labor supply. From equations (4) and (5), it is easy to find that spenders supply a fixed quantity of labor, i.e. $N_t^N = \frac{1}{1+\kappa}$.[2] Therefore, substituting spenders labor supply in (4):

(6) $$C_t^N = \frac{1}{1+\kappa}\frac{W_t}{P_t} = N_t^N \frac{W_t}{P_t}$$

The aggregate consumption and employment are

(7) $$C_t = \left(1-\lambda\right)C_t^R + \lambda C_t^N$$

(8) $$N_t = \left(1-\lambda\right)N_t^R + \lambda N_t^N$$

From aggregate equations (7)-(8) and (5), we obtain the aggregate labor supply:

(9) $$\frac{W_t}{P_t} = \frac{\kappa}{1-N_t} C_t$$

which states that the real wage is equal to the marginal rate of substitution between leisure and consumption.

[1] As usual the rule-of-thumb hypothesis is introduced as an *ad hoc* assumption. Here, for the sake of simplicity, we also assume that the savers are those that own the firms and demand money. Different assumptions may have different implication on short and long run income redistribution but do not affect our results.

[2] Note that employment of spenders does not rise in a demand-driven boom because we have assumed a logarithmic functional form for the consumer's instantaneous utility (see also Galì *et al.*, 2003; or Muscatelli *et al.*, 2003). A different form (e.g. constant relative risk aversion) eliminates the inelasticity of spenders' labor supply, but does not affect our main conclusions. Although this inelasticity is a drawback of the model, the logarithmic functional form greatly helps to simplify the exposition.

2. The supply side

2.1 The final sector

The supply side of the economy is composed by a continuum of firms producing differentiated intermediate goods, which are used as inputs by a perfectly competitive final goods-producing firm.

As usual, final good is produced by a perfectly competitive firm with the following technology:

(10) $$Y_t = \left[\int_0^1 Y_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}$$

where $Y_t(z)$ is the quantity of intermediate good z used as input by the representative firm.

The profit maximization yields the following set of demands for intermediate goods:

(11) $$Y_t(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\theta} Y_t$$

while from the zero profit condition we have:

(12) $$P_t = \left[\int_0^1 P_t(z)^{1-\theta} \right]^{\frac{1}{1-\theta}}.$$

2.2 The intermediate sector

Each intermediate goods firm is indifferent between the two types of households and produces output with a constant return to scale technology in a homogeneous labor input, $N_t(z)$, as follows:

(13) $$Y_t(z) = A_t N_t(z)$$

where A_t is an exogenous technology shock.

Given the constant return to scale technology and the aggregate nature of shocks, real marginal costs are the same across the symmetric intermediate good producing firms. Accordingly, from the cost minimization, we obtain the following labor demand:

(14) $$\frac{W_t}{P_t} = A_t \Phi_t$$

where W_t is the same across households.

2.3 Staggered prices adjustment

Following Calvo's setup firms adjust their price following a Bernoulli distribution. In each period firms have a constant probability, $(1-\varphi)$, to adjust their price, and a probability equal to φ to keep their price fixed. The time elapsing between every adjustment follows a geometric distribution, so that the expected waiting time for the next price adjustment is equal to $\frac{1}{1-\varphi}$.

Accordingly the problem of the firm changing price at time t consists of choosing $P_t(z)$ to maximize the following profits function:

$$(15) \qquad E_t \sum_{i=0}^{\infty} (\varphi\beta)^i \Lambda_{t,t+i} \left\{ \frac{P_t(z)}{P_t} Y_{t,t+i}(z) - \Phi_t Y_{t,t+i}(z) \right\}$$

subject to

$$(16) \qquad Y_{t,t+i}(z) = \left[\frac{P_t(z)}{P_t} \right]^{-\theta} Y_{t,t+i}$$

$$(17) \qquad \Lambda_{t,t+i} = \left(\frac{P_t C_t^R}{C_{t+i}^R} \right)$$

where $Y_{t,t+i}(z)$ is the firm demand function for its output at time $t+i$, conditional to its price setting i periods before, $P_t(z)$. $\beta\Lambda_{t,t+i}$ is the effective discount factor between time t and $t+i$.

From the first order conditions:

$$(18) \qquad P_t(z) = \frac{\theta}{\theta-1} E_t \sum_{i=o}^{\infty} \frac{(\varphi\beta)^i \Lambda_{t,t+i} Y_{t,t+i}(z)}{E_t \sum_{i=0}^{\infty} (\varphi\beta)^i \Lambda_{t,t+i} Y_{t,t+i}(z)} \frac{W_{t,t+i}}{A_{t,t+i}}.$$

The optimal price $P_t(z)$ is a markup, $\dfrac{\theta}{\theta-1}$, over a weighted average of expected future nominal marginal costs. In the symmetric equilibrium all adjusting firms finally choose the same price $P_t(z)$ and the same level of output $Y_t(z)$, so that the dynamics of the consumption-based index will be:

$$(19) \qquad P_t = \left[\varphi P_{t-1}^{1-\theta} + (1-\varphi) P_t(z)^{1-\theta} \right]^{\frac{1}{1-\theta}}.$$

2.4 The natural rate of output

Under flexible prices all the firms set their prices equal to a constant markup over marginal cost. Given the hypothesis of symmetric equilibrium, real marginal cost is constant and given by:

(20) $$\Phi_t = \frac{\theta - 1}{\theta}$$

Combining real marginal cost (14) with the aggregate labor supply $\frac{W_t}{P_t} = \frac{\kappa C_t}{1-N_t}$ yields:

(21) $$\Phi_t = \frac{\kappa C_t}{\left[1 - N_t\right] A_t} = \frac{\kappa Y_t}{\left[A_t - Y_t\right]}$$

where we use the definition of the aggregate production function, given by $Y_t = A_t N_t$, and the market clearing condition $Y_t = C_t$.

Combining (20) with (21) and solving for Y_t we obtain the following value for the "natural level of output," i.e. the flexible-price output:

(22) $$Y_t^n = \left(1 + \tfrac{\theta\kappa}{\theta - 1}\right)^{-1} A_t .$$

As expected, the "natural level of output" does not depend on monetary policy.

3. The log-linearized economy

The log-linearized model is obtained by considering the aggregate resource constraint ($Y_t = C_t$) and the demand and supply optimality conditions above derived.

Considering the demand side, by log-linearizing equations (3) and (7) we find:

(23) $$c_t^R = -\left(i_i - E_t \pi_{t+1}\right) + E_t c_{t+1}^R$$

(24) $$c_t = \left(1 - \lambda\right) \zeta_R c_t^R + \lambda \zeta_N c_t^N$$

Solving equation (24) for c_t^R yields:

(25) $$c_t^R = \frac{c_t - \lambda \zeta_N c_t^N}{\left(1 - \lambda\right) \zeta_R}$$

By using (25) and shifting it one period ahead into equation (23), we obtain:

(26) $$c_t = E_t c_{t+1} - \left(1 - \lambda\right) \zeta_R \left(i_i - E_t \pi_{t+1}\right) - \lambda \zeta_N E_t \Delta c_{t+1}^N$$

From the spenders' consumption function (6) we have:

(27) $$\Delta c_{t+1}^N = \Delta\left(w_{t+1} - p_{t+1}\right)$$

and thus the aggregate log-linearized Euler equation is

(28) $$c_t = E_t c_{t+1} - (1-\lambda)\zeta_R (i_i - E_t \pi_{t+1}) - \lambda \zeta_N E_t \Delta(w_{t+1} - p_{t+1})$$

Equation (28) can be rewritten as:

(29) $$c_t = E_t c_{t+1} - (1-\lambda\zeta_N)(i_i - E_t \pi_{t+1}) - \lambda \zeta_N E_t \Delta(w_{t+1} - p_{t+1})$$

since, from the aggregate equation (7), $(1-\lambda)\zeta_R = (1-\lambda\zeta_N)$ holds.

Equation (29) represents a modified version of the standard Euler equation, where i_t is the nominal interest rate. Consumption today depends on tomorrow expected consumption and on the real interest rate, but differently from the standard Euler equation, the presence of non-optimizing consumers establishes a link between the demand for goods and the real wage (see Galí *et al*. 2004; or Muscatelli *et al*. 2005; for further details). Notice that $\zeta_N = (1+\upsilon)\kappa(1+\kappa)^{-1}$, where $\upsilon = N(1-N)^{-1}$ is the inverse of the Frisch aggregate labor supply elasticity, which is independent of the spenders' share; in fact, by equating the supply and demand of labor (eqs. (9) (14)) and considering the aggregate resource constraint, we obtain $N_t(1-N_t)^{-1} = \Phi_t \kappa^{-1}$ that in the steady state is $\upsilon = \frac{\theta - 1}{\theta}\kappa^{-1}$ (see equation (20)).

By log-linearizing equation (9) and using the aggregate resource constraint, we obtain the aggregate labor supply:

(30) $$w_t - p_t = y_t + \upsilon n_t$$

Regarding the supply side of the economy, by log-linearizing (18) and using the definition (19), we obtain the familiar New-Keynesian Phillips-curve:

(31) $$\pi_t = \beta E_t \pi_{t+1} + \tau \phi_t$$

with $\tau = \frac{(1-\varphi)(1-\beta\varphi)}{\varphi}$. Equation (31) is a forward looking equation for inflation, which links movements of current inflation to contemporaneous movements in real marginal cost and expected inflation.

The log-linearization of the labor demand (14) is:

(32) $$w_t - p_t = \phi_t + a_t$$

The equilibrium of the labor market implies:

(33) $$\phi_t = y_t - a_t + \upsilon n_t$$

By considering equations (31) and (33). We obtain the New Keynesian forward relation between inflation and output:

(34) $\pi_t = \beta E_t \pi_{t+1} + \tau\left(y_t - a_t + \upsilon n_t\right)$

The log-linearization of the aggregate production (13) is:

(35) $y_t = a_t + n_t$

where a_t is assumed to follow a stationary first-order process $a_t = \rho^a a_{t-1} + \hat{a}_t$ with $\rho^a \in \left(0,1\right)$ and $\hat{a}_t \sim N(0,\sigma_a)$. Log-linearization of the flexible price output (22) leads to $y_t^n = a_t$, an increase in the technology shock, increases the natural rate of output. Thus:

(36) $x_t = y_t - y_t^n = y_t - a_t$.

Notice that, from (35) and (36), $x_t = n_t$ and that $\phi_t = \left(1+\upsilon\right) x_t$ (cf. eq. (33)).

Now we can express the log-linearized form of the model in terms of output gap. The model can be reduced to two familiar expressions. The IS relationship is derived from equations (29) and (30), by considering the aggregate resource constraint and the output gap definition (36) and rearranging:

(37) $x_t = E_t x_{t+1} - \Omega\left(i_t - E_t \pi_{t+1}\right) + \Omega \Delta a_{t+1}$

where $\Omega = \frac{1-\lambda\zeta_N}{1-(1+\upsilon)\lambda\zeta_N}$ is the income monetary multiplier.

The AS (Phillips curve) relationship is obtained from equations (34) and (36):

(38) $\pi_t = \beta E_t \pi_{t+1} + k x_t$

where $k = \tau\left(1+\upsilon\right)$.

The model dynamics is described by the following figure, where the impulse reaction functions to an AR(1) policy shock are plotted. The figure refers to the case λ = 0.6, the rest of the parameterization is standard, i.e. a degree of stickiness of 0.75, β = 0.99, as in Gali *et al*. (2004), labor disutility is set to obtain 1/3 employment, shock persistence is 0.9. The variable meaning is as follows: "co" is the consumption of the optimizer con summers; "i" is the nominal interest rate; "mu" is the markup (the inverse of the marginal cost), "n" is the aggregate employment (hours), "pi" is inflation and "ri" is the real interest rate.

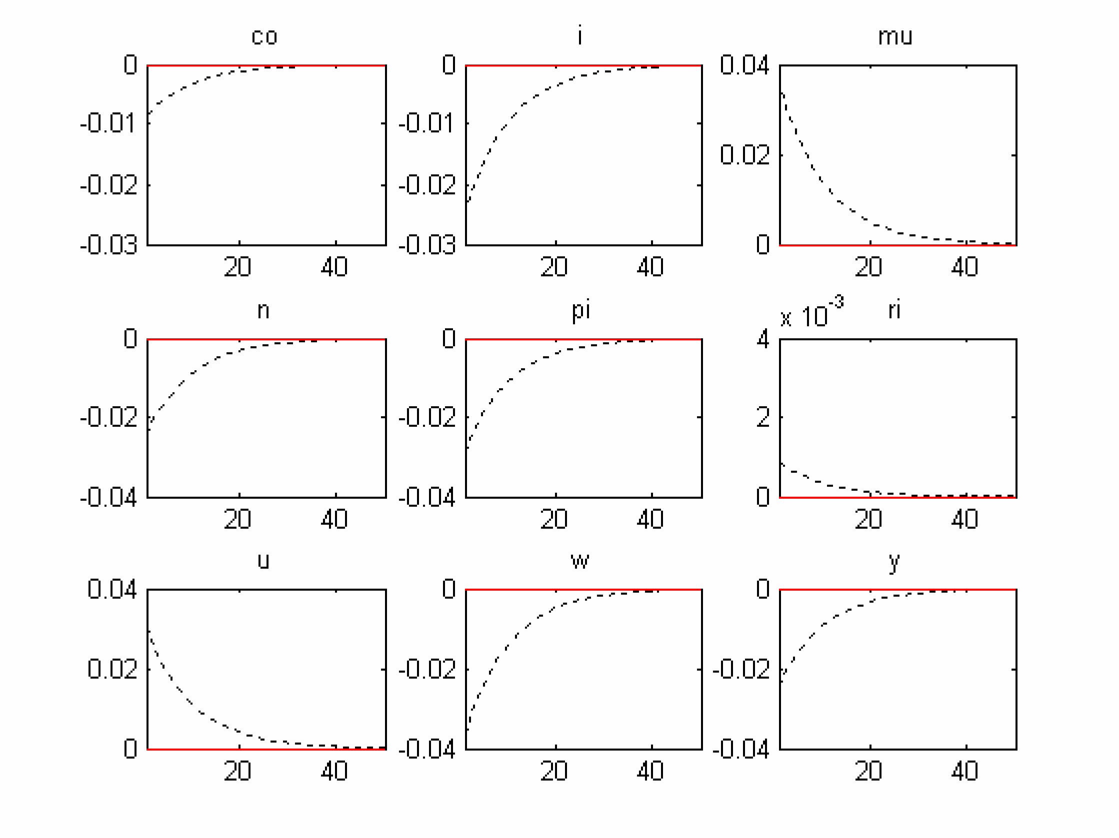

Figure 1

References

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