Taylor Rule under Rule-of-Thumb Consumers and External Habits: An International Comparison*

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Abstract. This paper develops and estimates a simple New Keynesian Dynamic Stochastic General Equilibrium (DSGE) model with rule-of-thumb consumers and external habits. Our theoretical model has a closed-form solution that allows us to derive its dynamical and stability properties. The model is also evaluated by running numerical simulations based on Monte Carlo Bayesian estimates of the structural parameters. The estimates are repeated for each of the G7 national economies. Providing single country estimates and simulations, we derive some indications on the relative efficacy of monetary policy and of its potential asymmetric effects resulting from the heterogeneity of the estimated model structures.

Keywords: Rule-of-thumb, habits, monetary policy transmission, determinacy, New Keynesian DSGE model, monetary policy, Monte Carlo Bayesian estimators.

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1. Introduction

Nearly fifteen years ago, Campbell and Mankiw (1989, 1990, 1991) provided evidence on the existence of heterogeneous consumers. The relevance of their findings is that they raised the question of a strong violation of the permanent income hypothesis. On the basis of their results, only a fraction of households is able to plan consumption according to the standard Hall’s consumption function, (Savers), while there is a relevant fraction of households that equates current consumption to the current income period by period (Spenders).\(^1\) The theoretical and policy implications of consumer heterogeneity are rather strong. Considering the fiscal policy theory, an important consequence is that the Barro-Ricardo equivalence theorem does not hold if Spenders are considered. For this reason, savers are often referred to as Ricardian consumers and Spenders as Non-Ricardian consumers (Mankiw, 2005).

The role played by Spenders has been more recently analyzed with respect to its implications for monetary policy in New Keynesian theoretical frameworks (Amato and Laubach, 2003; Gali et al.,

\(^1\) Spenders’ behavior can be interpreted in various ways. One can view their behavior as resulting from consumers who face binding borrowing constraints. Alternatively, myopic deviations from the assumption of fully rational expectations should be assumed (rule-of-thumb), i.e. consumers naively extrapolate their current income into the future, or weigh their current income too heavily when looking ahead to their future income because current income is the most salient piece of information available. See Mankiw (2000) and references therein.
The general finding is that the presence of Spenders’ may alter dramatically the properties of these models and overturn some of the conventional results found in the literature. Amato and Laubach (2003) explore the optimal monetary rule with rule-of-thumb households and firms. By modeling consumers’ rule-of-thumb behavior as a consumption habit, households’ current decisions mimic past behavior of all agents (including optimizing agents). They also show that, while the monetary policy implications of rule-of-thumb firms are minimal, the optimal interest rate is sensitive to the presence of rule-of-thumb consumers; as their fraction increases, higher inertial monetary policy is required.

Gali et al. (2003) show how the Taylor principle becomes a too weak criterion for stability when the fraction of rule-of-thumb consumers is large. However, the presence of Spenders cannot in itself overturn the conventional result on the sufficiency of the Taylor principle. By contrast, in the case of forward-looking interest rate rules, they show that the conditions for a unique equilibrium are somewhat different from those emerging in a context of contemporaneous rules. In particular, they demonstrate that when the share of Spenders is sufficiently large it may not be possible to guarantee a (locally) unique equilibrium or, if it is possible, it may require that interest rates respond less than one-for-one to changes in expected inflation. Bilbiie (2005) discusses the implications of limited asset market participation for optimal monetary policy, from both a theoretical and empirical point of view. His main point is that when limited asset market participation is considered a passive interest rate rule is consistent with a welfare-maximizing monetary policy.

The aim of this paper is twofold. First of all, we provide the dynamic and stability properties of a model in which both rule-of-thumb consumers and external habits in Savers’ consumption are considered. Second, on the basis of country-specific estimates of the structural parameters, we evaluate the empirical relevance of our hypotheses and the potential heterogeneity of the effects of the monetary policy.3

3 For the country-specific empirical analysis we consider the single G7 economies.
By considering a simple setup without capital accumulation, we can derive a closed-form solution. This allows us to analytically discriminate, on the basis of the fraction of Spenders, between two different demand regimes (i.e. two IS-curves) characterized by sign inversion of the coefficient capturing the correlation between the real interest rate and expected consumption growth. The possibility of a demand regime shift has a dramatic importance for the analysis of monetary policy efficacy. Moreover, it has serious implications for the analysis of equilibrium determinacy, as discussed in Bilbiie (2004 and 2005) and Di Bartolomeo and Rossi (2005). With respect to the efficacy of monetary policy, it has been shown that, if the correlation between expected consumption growth and real interest rates is positive, the efficacy of monetary policy increases in the fraction of Spenders (Amato and Laubach, 2003, Di Bartolomeo and Rossi, 2005). A reverse result is instead obtained if the correlation between expected consumption growth and real interest rate is negative (Di Bartolomeo and Rossi, 2005). Concerning determinacy, we find that, in the case of a positive correlation, standard results hold, i.e. if monetary policy follows a standard Taylor rule determinacy is always associated to the satisfaction of the Taylor principle. By contrast, if the correlation is negative, we find different requirements for stability, that are conditional on the magnitude of the effects of interest rates changes on the real output. Hence, the non-conventional results stressed by Gali et al. (2003) hold only if the correlation between expected consumption growth and real interest rate is negative. The joint consideration of habit persistence in consumption and of Non-Ricardian consumers increases the complexity of the model, since the monetary multiplier becomes highly nonlinear. Employing numerical simulations we show that the introduction of habits, by increasing the threshold fraction of Spenders for which a sign inversion of the monetary multiplier can be obtained, reduces the probability of obtaining a regime shift in the demand schedule.

A further motivation for the recent interest on the introduction of Spenders into the New Keynesian DSGE model is its potential empirical relevance. The presence of Spenders may in fact explain the puzzling result of a negative correlation between expected consumption growth and real interest
rate. The correlation between output changes and the real interest rate has been found to be low and sometimes negative across many of the industrialized countries that have been considered in the empirical investigations (see Ahmad, 2004, Canzoneri et al., 2002).

The empirical evaluation of the aspects outlined above is particularly important for analyzing the potentially heterogeneous effects of monetary policy shocks among countries. The importance of these potential drawbacks is evident in the fact that the EMU countries face a centralized monetary policy. The empirical relevance of the different theoretical predictions is however still ambiguous. First of all, to the best of our knowledge, there are only few studies that have empirically addressed the effects of deviations from Hall’s benchmark consumption for the forward-looking IS relation. Moreover, even when the problem has been explicitly or implicitly considered, the results obtained are weak and substantially inconclusive. In a recent paper, Fuhrer and Olivei (2004) provide empirical evidence for the parameters of a reduced form IS equation, defined in a standard New Keynesian model augmented with habits. The estimated income monetary multiplier resulted weakly negative or not significant in statistical terms. Bilbiie (2005) explicitly deals with the question of the monetary policy implications of the presence of relevant liquidity constraints in consumption behavior. Even if the paper addresses specific issues regarding the empirical relevance of the liquidity constraints, the evidence cannot be considered conclusive. First, the main purpose of the work is to give an evaluation of the monetary policy conduct, in the pre-Volker and pre-financial liberalization period and not a direct estimate of the fraction of rule-of-thumb households. Second, even if the analysis is based on empirical results, these results cannot be considered particularly informative for our purposes. They are in fact obtained estimating a reduced form IS relation, which coefficients are only a convolution of the (structural) parameters of interest.

4 A direct consideration of these aspects can be found in Bilbiie (2005), who shows how the introduction of Spenders into the analysis can justify an alternative explanation of the evolution of the American monetary policy in the pre and post Volker period. He argues that the different reaction function of monetary policy derives from a different development of the financial markets, i.e. from modifications in the fraction of spenders. In this context, monetary policy is optimal in both periods and does not lead to indeterminacy. Bilbiie (2005) also provides some empirical evidence supporting his point of view.

5 Moreover, their empirical relevance is potentially flawed, since results are obtained employing a GMM estimator, which small sample performances have been shown to be dramatically poor. A further problem with GMM estimation
In our empirical investigation we are mainly interested to the assessment of the potential heterogeneity of the effects of monetary policy. The efficacy of the monetary policy and its sensitivity to different parameterizations are evaluated simulating DSGE models that are parameterized employing the single countries’ estimates of the structural coefficients. Thus, the values of the deep parameters are not calibrated or fixed on the basis of previous evidence, as in the standard practice. Our strong empirical stance suggests of estimating the structural coefficients employing quarterly data for the seven most industrialized economies (G7) for the period 1963-2003. Differently from the common practice emerging in recent studies (Smets and Wouters, 2003, Coenen and Straub, 2005), we consider these data separately in order to stress the cross-country heterogeneity. The complexity and nonlinearity of the resulting structure of the model suggests the implementation of a Bayesian Monte-Carlo Markov Chain estimation procedure (MCMC). Once the different sets of structural parameters are obtained on a single country basis, the resulting structures are simulated, in order to appreciate the different responses to typical shocks, in particular to monetary policy shocks. As it will be stressed with more detail in the following, our analysis is close in spirit to the strategy proposed by Smets and Wouters (2003) for the estimation of their New Keynesian model. The main differences with respect to their analysis are that we do not consider capital accumulation and that we introduce Non-Ricardian consumers. Moreover, we develop a single country analysis for the G7 economies, while their estimates are referred to aggregate data of the Euro area.

The rest of the paper is organized as follows. Section 2 outlines the basic framework and describes the two demand regimes implied by the presence of Spenders and Savers; it further discusses the model properties by closely examining the rational expectation equilibrium determinacy and the transmission mechanism of monetary policy. Section 3 presents an empirical examination of the model employing the relevant data of the seven major economies (G7) and is organized as follows:

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is that the chances of finding a theoretically consistent instrumentation for the moment conditions are strongly reduced when the number of modifications imposed to the standard consumption function increases. This problem is of particular relevance if we consider that our theoretical framework includes both habits and rule-of-thumb consumption.
the first part contains the description of the Bayesian MCMC estimation of the structural parameters of the model and the discussion of the simulation results obtained with the estimated structures. Section 4 concludes.

2. The basic theoretical framework

2.1. The model

We develop a simple New Keynesian model augmented with Non-Ricardian consumers and habit formation. In order to simplify the analysis and highlight the demand-side effects of Spenders’ behavior we do not consider the capital accumulation process. We assume a continuum of infinitely-lived heterogeneous agents normalized to one. A fraction \(1 - \lambda\) of them consumes and accumulates wealth as in the standard setup (Savers). The remaining fraction \(\lambda\) is composed by agents who do not own any asset, cannot smooth consumption, and therefore, consume all their current disposable income (Spenders). We also assume that Savers consumption at time \(t + i\) depends on habits inherited from past consumption, i.e. on a fraction \(\gamma\) of lagged aggregate consumption. Representative consumers are indexed by \(R\) (Ricardian or Savers) and \(N\) (Non-Ricardian or Spenders). At the date zero, they maximize the following functions:

\[
E \sum_{t=0}^{\infty} \beta^t u \left( C_{t+i}, C_{t+i-1}, N_{t+i}, \phi^j \right) \quad j \in \{R, N\}
\]

where \(\beta \in (0,1)\) is the discount factor, \(C_t\) is household consumption at time \(t\), while \(N_t\) is labor supply. \(\phi^j\) is a binary variable such that when \(j = R\), \(\phi^R = 1\) and when \(j = N\), \(\phi^N = 0\). For sake of simplicity, we use a logarithmic utility function, which allows us to obtain a closed-form solution of the model. We assume the following instantaneous utility,

\[
u(.) = \ln \left( C_{t+i} - \gamma \phi^j C_{t+i-1} \right) - \kappa \ln \left( 1 - N_t \right),
\]

with \(0 \leq \gamma \leq 1\) (habit coefficient), and \(\kappa > 0\). In addition, the following budget constraints hold:
where $W_t$ is the nominal wage at time $t$, $\Pi_t$ is profit sharing. Real wages are the only source of Spenders’ disposable income; therefore, they are subject to a static budget constraint, while Savers face the standard dynamic constraint. Since Spenders do not save, they consume all their current income.

By solving the inter-temporal optimization problems of Savers and Spenders, aggregating and then log-linearizing, we obtain the following description of the demand side of the economy:

$$(3) \quad c_t = - \frac{1-\sigma - \lambda N}{1+\sigma} (i_t - E_{\pi_t+i_t}) + \frac{1}{1+\sigma} E_t c_{t+1} + \frac{\sigma}{1+\sigma} c_{t+1} - \frac{\lambda N}{1+\sigma} E_t (w_{t+1} - p_{t+1})$$

$$(4) \quad w_t - p_t = \nu n_t + (1-\sigma) c_t - \sigma (1-\sigma) c_{t+1}$$

where $c_t$ is consumption, $i_t$ is the nominal interest rate, $\pi_t$ is the inflation rate and $w_t - p_t$ is the real wage. Since only Savers have consumption habits, $\omega = \gamma (1-\lambda)$ is the “aggregate” habit parameter; $\nu = N (1-N)^{-1} = \theta \kappa^{-1} (1-\sigma)^{-1}$ is the inverse Frisch elasticity, while $\theta = (\eta-1)\eta^{-1} \in (0,1)$ is the mark-up, which is a function of the elasticity of substitution of intermediate goods $\eta$; $\kappa$ indicates labor disutility and $\zeta^N = \kappa (1+\kappa)^{-1} (1+\nu)(1-\sigma)$ is the share of Spenders’ consumption at the steady-state.

Equation (3) is a modified version of the standard consumption Euler equation, while equation (4) is the consumers’ aggregate labor supply. Our Euler equation (3) differs (qualitatively) from the standard habit formation version, because it depends on expected changes in the real wage. This is due to the presence of Savers, which establishes a link between the demand for goods and the real wage.

Given the economy production function, $y_t = a_t + n_t$, the resource constraint, $y_t = c_t$ and equation (4), equation (3) can be expressed as a modified IS-curve:
which is apparently similar to the standard New-Keynesian representation of the demand side of the economy. The next subsection will discuss equation (5) in more detail.

In the New-Keynesian framework, the supply side of the economy is described by a continuum of firms producing differentiated intermediate goods for a perfectly competitive final goods market. Intermediate firms cannot adjust their prices every period; conversely, each period they face a certain probability of being able to do it (the Calvo’s lottery). In setting their price, firms thus take into account future marginal costs (by considering inflation expectations) in addition to the current marginal cost. As a result, the price-adjustment mechanism is described by the following forward-looking relationship:

\[ (\pi_t = \beta E_t \pi_{t+1} + \tau mc_t, \] 

where \( \tau = (1-\phi)(1-\beta \phi)\phi^{-1} \). The parameter \( \phi \) defines price staggering, i.e. the fraction of firms maintaining their price fixed each period. Since labor is the unique input of the intermediate sector and the production function is linear, the sticky-price equilibrium real marginal costs are:

\[ mc_t = \frac{1+\nu(1-\sigma)}{1-\sigma} y_t - \frac{\sigma}{1-\sigma} y_{t-1} - (1+\nu) a_t. \]

Since we assume that the markup is constant at the steady-state, the flexible-price equilibrium log-linearized real marginal costs are zero. Substituting (7) in (5) and solving for \( y_t \), we obtain the natural rate of output, defined as the output under flexible-price equilibrium \( y_t' \),

\[ y_t' = \frac{(1+\nu)(1-\sigma)}{1+\nu(1-\sigma)} a_t + \frac{\sigma}{1+\nu(1-\sigma)} y_{t-1}'. \]

The flexible-price output is a weighted average of technology and of its past value. The inertial component of output is increasing in the aggregate habit parameter and decreasing in the inverse
Frisch elasticity. Thus, the introduction of rule-of-thumb consumers reduces the role played by the inertial component in the natural rate of output adjustment process since it reduces the aggregate habit parameter. If habit persistence is not present, equation (8) collapses to the standard natural output equation.

From equations (6), (7) and (8) we can easily derive the so-called New Keynesian Phillips curve:

\[ \pi_t = \beta E_t \pi_{t+1} + \frac{\tau (\kappa + \theta)}{\kappa (1 - \sigma)} (y_t - y_t^f). \]

Notice that if we assume nonzero habit persistence in consumption, the fraction of Non-Ricardian consumers affects the coefficient for the inflation response to the output gap, otherwise it has no role.

The model can be summarized by three equations: equations (5) and (9), which respectively describe the demand (IS curve) and supply (New Keynesian Phillips curve) side of the economy, and equation (8), which defines the flexible-price natural rate of output. The introduction of rule-of-thumb consumers crucially affects the first of the above relationship that, in turn, crucially affects the properties of the model, as we are going to show in the next subsections.

\[ \Omega = \frac{\partial y}{\partial i}. \]  

\[ \text{Equation (5) can thus individuate two different demand regimes:} \]

\[ ^6 \text{Bilbiie (2004), in a similar model, finds that, for high value of rule-of-thumb consumers, the slope of the IS curve changes in sign so that an interest rate increase becomes expansionary; he also tests the existence of the two different regimes using U.S. economy data. Differently from Bilbiie we study the theoretical effect on the slope of the IS curve adding the hypothesis of habits persistence and empirically testing the model looking for both qualitative and quantitative difference across the G7 countries.} \]
1. A standard demand regime – i.e. a negatively sloped IS curve – holds if the interest rate elasticity is negative. Such a regime is coherent with the hypothesis of life-cycle permanent income and thus with the consumption smoothing theory;

2. An inverse demand regime – i.e. a positively sloped IS curve – holds if the interest rate elasticity is positive. In other words, the demand regime is dominated by the liquidity-constraint effect, for which an increase in the real interest rate is expansionary while interest rate cuts imply demand contractions, since many consumers cannot access to financial markets and saving.

We first discuss the case without habits, i.e. $\gamma = 0$ or $\sigma = 0$. In this case $\zeta^N$ and $\nu$ are independent of the Non-Ricardian consumer’s fraction and $y^f = a_i$. It is straightforward to show that the two regimes depend on a threshold value of $\lambda^*$. The traditional regime holds for:

$$\lambda < \lambda^* = \frac{1}{\zeta^N(1+\nu)} = \frac{\kappa(1+\kappa)}{(\kappa+\theta)^2}.$$ 

If the inequality (12) is not satisfied, the liquidity-constraint regime emerges. For relatively low values of $\theta$ and high values of $\kappa$, the threshold value is greater than one ($\lambda^* > 1$). In such a case, only the standard regime occurs since $\lambda \in [0,1]$. In other terms, the inverse Frisch elasticity is smaller than one. For relatively high values of $\theta$ and low values of $\kappa$, the liquidity-constrained regime can emerge. In addition, if $\theta$ is greater than 0.5, $\lambda^*$ is always smaller than one. Thus, in such a case, the liquidity-constraint regime can emerge for sufficiently high values of $\lambda$.

Introducing habit persistence in the model, the interest rate elasticity becomes highly non linear in $\lambda$ since $\zeta^N$, $\nu$ and $\sigma$ depend on it. Thus, the analytical derivation of the conditions for regime shifts becomes problematic. An implicit condition can be derived:

$$\lambda < \frac{(1-\sigma^2)\kappa(1+\kappa)}{\kappa(1-\sigma)(1-\sigma^2) + \theta[\kappa(1-\sigma) + \theta]}$$

From the expression above it is clear that as $\theta$ increases the inverse regime is more likely to be
observed. The effects of $\kappa$ are more ambiguous; numerical simulations\(^7\) show that for high values of $\kappa$ the inverse regime is never observed. Numerical simulations also show that, other things equal, by augmenting the habit parameter the threshold value of $\lambda$, necessary to obtain the monetary multiplier sign inversion tends to increase; hence, the probability of observing a demand regime shift decreases.

2.3 Demand regimes and equilibrium determinacy

The recent literature on central banking has shown that one of the fundamental tasks of the monetary authority is to support rational expectation equilibrium determinacy. In order to close the model and study its determinacy properties, we consider a simple feedback Taylor rule:\(^8\)

\[(12) \quad i_t = \alpha_1 \pi_t + \alpha_2 y_t + k_t,\]

where $\alpha_1$ and $\alpha_2$ are positive parameters and $k_t$ is a constant or a stochastic term (representing a stationary disturbance process), which does not affect the conditions for determinacy.

Determinacy crucially depends on two factors: the demand regime and monetary policy efficacy. These factors can be identified by the sign and the size of the elasticity of income to the interest rate. A negative (positive) sign occurs in the standard (inverse) regime; efficacy increases in the elasticity modulus, i.e. $|\Omega|$.

In the standard regime determinacy under a feedback Taylor rule requires an active policy rule satisfying:

\[(13) \quad a_t > 1 - \frac{1 - \beta}{k} a_z,\]

where $k = \frac{\sigma(\kappa + \sigma)}{\sigma^2(1 - \sigma)}$ is the elasticity of the price adjustment with respect to the real output (see

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\(^7\) Results of our simulations and their sensitivity analysis are reported in the longer working paper version of our article.

\(^8\) John Taylor has proposed that U.S. monetary policy in recent years can be described by an interest-rate feedback rule as that considered here (see Taylor, 1993). Note also that the feedback Taylor rule can also be used to study determinacy of an endogenous policy derived from the so-called flexible inflation targeting approach (Svensson, 1999; Evans and Honkapohja, 2005) or from the utility-based welfare maximization (Woodford, 2003: Ch. 6). We discuss and derive the condition of this case in the working paper version of the article. See also Di Bartolomeo and Rossi (2005).
equation (9)). The condition has the usual interpretation. A feedback rule satisfies the Taylor principle if in the event of a sustained increase in the inflation rate by one percentage point, the nominal interest rate will be raised by more than one percentage point. Each percentage point of permanent increase in the inflation rate implies an increase in the long-run average output gap of \((1 - \beta)k^{-1}\) percent. An exogenous Taylor rule thus conforms to the Taylor principle if and only if its coefficients satisfy \(a_i + (1 - \beta)k^{-1}a_z > 1\) (see Woodford, 2004).

In the liquidity-constrained regime, determinacy requires (see Appendix A):

\[
(14) \quad a_i > \max \left\{ 1 - \frac{1 - \beta}{k} a_z, \frac{2}{\Omega} a_z - a_z \right\} \frac{1 + \beta}{k} - 1 \} \text{ or }
\]

\[
(15) \quad a_i < \min \left\{ \frac{1 + \beta}{k} a_z - a_z, 1 - \frac{1 - \beta}{k} a_z, \frac{2}{\Omega} a_z - a_z \right\} \frac{1 + \beta}{k} - 1 \}
\]

Two situations arise according to the efficacy of monetary policy. If \(\Omega > (1 + \beta)k^{-1}\), the Taylor principle (13) leads to determinacy, but the equilibrium is also stable if \(a_i < \min \left\{ \frac{1 + \beta}{k} a_z - a_z, \frac{2}{\Omega} a_z - a_z \right\} \frac{1 + \beta}{k} - 1 \}. \) By contrast, if \(\Omega < (1 + \beta)k^{-1}\), stability requires either \(a_i > \left( \frac{2}{\Omega} a_z - a_z \right) \frac{1 + \beta}{k} - 1\), i.e. a more aggressive policy than the Taylor prescription (13), or \(a_i < \min \left\{ \frac{1 + \beta}{k} a_z - a_z, 1 - \frac{1 - \beta}{k} a_z \right\} \frac{1 + \beta}{k} - 1 \}\).

Summarizing our findings, in the standard regime the Taylor principle provides the necessary and sufficient conditions for determinacy. Differently, in the liquidity-constrained regime, if monetary policy has a relative high efficacy \((\Omega > (1 + \beta)k^{-1})\), the Taylor principle is a sufficient but not necessary condition for determinacy since also a (relatively) loose policy leads to the same result. By contrast, if monetary policy has a relatively low efficacy \((\Omega < (1 + \beta)k^{-1})\), the Taylor principle does not leads to determinacy; a sufficient condition for determinacy requires a stronger reaction to inflation or again a loose policy.
2.4. Monetary policy efficacy and its transmission mechanisms

Different demand regimes, defined as above, may imply different policy regimes, i.e. heterogeneous transmission mechanisms of the monetary policy. The emersion of different policy regimes is related to both the sign of the demand elasticity to the real interest rate (demand regime) and to its size (monetary policy efficacy). Consider that if the monetary policy is set according to a Taylor rule of the kind (12) augmented with a white noise shock (i.e. a monetary policy disturbance), three different policy regimes can be individuated (see Appendix B).

1. In the standard demand regime ($\Omega < 0$), as usual, a positive interest rate disturbance i.e. an increase in the nominal interest rate, always has deflationary effect by decreasing inflation and real activity. The real interest rate increases.

2. In the inverse regime ($\Omega > 0$) two different situations can occur:

(a) If $\Omega > (a_1k + a_z)^{-1}$, a positive monetary disturbance will affect the economic outcomes as in the standard regime even if the elasticity of the aggregate demand with respect to the real interest rate is positive, but the real interest rate is moving in the opposite direction of the nominal one.

(b) By contrast, if $\Omega < (a_1k + a_z)^{-1}$ a positive monetary shock implies an increase in the output gap and inflation. The real interest rate falls. Note that this regime is potentially compatible with the price puzzle, i.e. (apparent) positive empirical relationship between the federal funds rate and inflation (Sims, 1992).

The rationale of the two non-standard policy regimes (a) and (b) can be explained as follows. In the inverse regime a negative monetary shock initially shifts the aggregate demand backwards and, thus, it reduces real output and inflation. The lower real output and inflation stimulate the expansionary central bank’s reaction which, however, eventually leads to either an increase or a reduction of the real interest rate. The emerging policy regime depends on the monetary policy efficacy. Neglecting the importance of habit formation, the efficacy of monetary policy is
increasing in the fraction of Spenders in the standard demand regime. By contrast, in the liquidity-constrained regime, its efficacy is decreasing in the fraction of Spenders. By introducing habits, the transmission mechanics becomes more complex and numerical simulations are needed. We will discuss it more in detail after our empirical investigation.

3. Bayesian MCMC estimation of the structural parameters

3.1 Data presentation and a brief description of the estimation approach

The sample employed for our estimates is composed of quarterly time series for GDP, GDP deflator, labor compensation, employment, and nominal interest rates. The time period covered by the sample information is 1963:1 to 2003:2 and we consider G7 countries. In the benchmark formulations, we employ short term nominal interest rate definitions such as the Federal Funds Rate for the United States, the Overnight Rate for Canada and the United Kingdom and the Money Call Rate for the remaining countries. In order to check for robustness, we also re-run the estimations by substituting the reference short-term rates with the three months Treasury Bill Rate and the 10-years Government Bonds Rate. Data are all drawn from the IMF International Financial Statistics (IFS) database. The log real output gap is obtained as the difference between log real output and its trend, the latter estimated employing the Hodrick-Prescott filter. Following Smets and Wouters (2003), real output is de-trended assuming a linear trend while, because of their co-trending behavior, both inflation and the nominal interest rate are de-trended on the basis of the estimated linear component in inflation.

As regards the approach, since we are interested to the estimation of the structural parameters of the theoretical model described in the previous section, the resulting computational task is somewhat complicated, as the strong nonlinearity of the model may significantly affect the performances of the numerical methods for unconstrained Full Information Maximum Likelihood (FIML)
estimation.\(^9\) Even if a viable solution would be to restrict the parameters to assume values within a defined range that we deem as \textit{reasonable}, we adopt a Bayesian Monte Carlo approach.

The Bayesian approach implemented here\(^{10}\) is close in spirit to restricted FIML estimation. Instead of employing interval restrictions on FIML estimated parameters, we use a procedure which nests a formalized distributional \textit{a priori} on parameters with the conditional distributions – i.e. with the likelihood function – in order to obtain a posterior density that we will consider as the benchmark distribution for our Monte Carlo parameter estimates. The final estimates will be obtained employing the Metropolis-Hastings procedure implemented in Dynare for Matlab (Juillard, 2004).

The posterior density is a result of a weighted average of the prior distributions and of the likelihood function (i.e. the empirical information), with weights inversely related to, respectively, the variance of the prior distributions and the variance of the sample information (“precisions”). The bigger the informative power of the likelihood (i.e. the lesser the variances of the likelihood-based parameters), the closer the posterior will be to the conditional distribution. In the limiting case in which the data allow a perfect knowledge of the parameter values, the posterior density collapses to the conditional distribution. Contrary, if the empirical information is weakly informative, the priors will correspondingly have more weight in the estimation. Formalizing a tight prior will result in highly constrained estimation, while assuming a diffuse prior will result in weakly constrained estimation.

Formally, our procedure requires of nesting the prior distribution \(P(\theta)\) for the parameter vector \(\theta \in \Theta\) and the conditional distribution (or likelihood)\(^{11}\) \(P(Y_T \mid \theta),\ Y_T = \{y_t\}_{t=1}^T\) to get the posterior distribution \(P(\theta \mid Y_T)\). Basically, this is obtained employing the Bayes rule, i.e.

\[
P(\theta \mid Y_T) = \frac{L(Y_T \mid \theta)P(\theta)}{P(Y_T)},
\]

\(^{11}\) The conditional distribution is obtained employing the Kalman filter (Sargent, 1989).
where \( P(Y_t) \) is the marginal distribution. Once the posterior distribution is obtained, it will be employed as the “proposal density” to initialize the M-H MCMC sampling method,\(^{12}\) which substantially generates a large number of random draws from the posterior density in order to obtain a Monte-Carlo estimate of the parameters’ means and distributions.

Operationally, the model is estimated employing four observable variables, log real private output, first differences of the log GDP deflator, the quarterly nominal interest rate and a measure of the log real output gap, obtained as explained above.

### 3.2 Structure of the model and prior distributions

On the basis of the derivations made in section 2, for the purpose of estimation and simulation we consider the log-linear system defined by equations 5 to 8 augmented with a Taylor-like monetary policy reaction function, an output gap definition and five structural shocks.

The operational structure is summarized in the following box.

About here Box 1.

The meaning of the equations is intuitive. The first equation is the IS relation, in which the four reduced form coefficients \( \Omega_i, i = 1,\ldots,4 \), include the corresponding structural parameters in equation (5), the second equation is the expectation-augmented Phillips curve and the third equation is a Taylor-like rule in the spirit of that employed by Smets and Wouters (2003). The fourth equation is the marginal costs definition under rule-of-thumb and habit persistence, the fifth the output gap definition and the sixth defines the process for natural output. The last three equations define the autoregressive processes for the three permanent components of our model.

For the monetary policy reaction function we assume autoregressive interest rate smoothing, which intensity is defined by the parameter \( \rho_i \), and that the monetary authorities react to deviations from targeted inflation \( \pi^*_t \) (assumed to be zero at the beginning of the simulation) and to the output gap

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\(^{12}\) More precisely, the algorithm employs the mode and the Hessian evaluated at the mode for the initialization of the M-H procedure.
The structural shocks hitting the economy are: i) a preference shock $\varepsilon^{p}_{t}$, ii) a technology shock $\varepsilon^{a}_{t}$, iii) a cost-push shock $\varepsilon^{cp}_{t}$, iv) a monetary policy shock $\varepsilon^{i}_{t}$, and v) a shock on the monetary policy target, i.e. on targeted inflation, $\varepsilon^{\pi}_{t}$. We also assume that the preference, the technology and the monetary policy target shocks are somewhat persistent, giving rise to autoregressive stationary processes governing preferences, technology and targeted inflation. The other shocks are represented by serially uncorrelated i.i.d. innovations. This characterization of the shocks is needed in order to reproduce the persistence and hump-shaped responses found in the data. It represents a quite weak assumption from a theoretical point of view, as it is commonly accepted that technology shocks, as well as preference shocks, have generally long-lasting effects, while the permanence of the monetary policy target can be justified on the grounds that, once convinced and committed on a given target, authorities change their mind slowly.

The shape of the prior distributions is chosen according to the following assumptions: we assume, as in Smets and Wouters (2003), that the reference distribution for the structural shocks is the inverted gamma distribution with two degrees of freedom, which is consistent with a diffuse prior on perturbations and positive variances; for parameters theoretically defined in a 0-1 range, we assume a beta distribution, while for the other parameters we assume a normal distribution. The means and standard deviations are chosen on the basis of the empirical reliability of the information obtainable from other studies or from the results of preliminary GMM and ML estimates conducted on reduced-form equations for the seven countries.

Differently from Smets and Wouters (2003), since in our log-linear formulation of utility the dimensionality of the parameterization is reduced, with the exception of the discount factor $\beta$ which is fixed at 0.995 (this is consistent with a steady state real rate of 2%), we do not employ fixed parameters values. Anyway, we adopt relatively tight priors for the elasticity of substitution across intermediate goods $\eta$ and for labor disutility $\kappa$.

As a result of the model assumptions described above, we have to estimate 16 parameters, of which
five define the distribution of the structural shocks and three their persistence.

Concerning prior mean values, in line with Gali et al. (2003), the expected elasticity of substitution across intermediate goods \( \eta \) is set to 6, which is consistent with a steady-state mark-up of 20%. The mean of the labor disutility parameter \( \kappa \), set to 3, is chosen on the basis of the ratio between hours spent at work and total available time. For both parameters, we assume a relatively small prior variability of, respectively, 0.3 and 0.15, and a normal prior shape, consistent with a 5% coefficient of variation. For the Taylor rule parameters we assume that the mean values for the parameter on expected inflation and for the parameter on output gap are, respectively, 1.5 and 0.125. Prior standard deviations are, respectively 0.15 and 0.05 and the prior shape of the distribution is again the normal. The chosen variability implies a moderately diffuse prior for the first parameter and a very diffuse prior for the second parameter. These values are also consistent with the average ML estimates of the Taylor rule parameters conducted for the seven countries included in the analysis. The prior mean of the interest rate smoothness parameter, consistently with the average ML estimates, is 0.8, while for its variability we assume a prior of 0.10, which can be considered relatively large with respect to the empirical standard deviations found with the unconstrained ML estimates. The chosen prior shape for the distribution of the interest rate smoothness parameter is the beta distribution.

For the fraction of firms maintaining the price fixed \( \phi \) we assume a prior mean of 0.75, which is consistent with the results of Gali at al. (2001). These authors obtained an average duration of the price contracts of approximately one year and a rather small prior variability, consistent with a range between 3 and 6 quarters.

For the parameters characterizing the persistence of shocks we adopt a common mean value of 0.85 and a prior variability of 0.10. As in Smets and Wouters (2003), the choice of a relatively concentrated prior for the persistence parameters is justified by the need of having a tight separation between persistent and transitory shocks, which enhances the identification of the two shocks entering the interest rate equation. The prior shape is the beta distribution.
For the habit persistence parameter we assume a prior mean value of 0.7 associated with a moderately diffuse prior variability of 0.1. The shape of the prior distribution is again the beta distribution. Prior mean and variability are chosen on the basis of the evidence emerged in a number of previous studies and on the basis of the results of our GMM estimates of the parameters of an Euler equation for consumption, modified in order to account for habit persistence.

For the rule-of-thumb parameter we set a prior mean of 0.5 and a prior S.D. of 0.10, while the reference distributional shape is again the beta. These prior values are consistent with the findings of Campbell and Mankiw (1989) and with the average result from our GMM estimates on the seven major economies data.

For the structural shocks we basically adopt a parameterization which is similar to that employed by Smets and Wouters (2003). Apart from the large interval implied by the assumption of two degrees of freedom for the inverted gamma distribution, the prior mean values are obtained from previous estimations conducted with very diffuse priors.

The table below reassumes the prior distributions for the structural parameters considered in the analysis.

Simulations based on the prior calibration gives encouraging results. The simulated moments of the artificial series are close to the empirical moments of the U.S., and the impulse responses are qualitatively consistent with the theoretical expectations. The typical hump-shaped behavior found with VAR-based empirical impulse responses is well reproduced both in the extent and in the duration of the deviations from steady-state equilibrium.

---

13 E.g. Fuhrer (2000) finds that about one-fourth of income accrues to rule-of-thumb consumers in the United States. Muscatelli et al. (2003) find an even larger proportion. They suggest that about 37% of consumers are rule-of-thumb consumers, whilst 84% of total consumption in steady state is given by optimizing consumers. Rule-of-thumb consumers account for about 59% of total employment. Additional evidence is provided by Jappelli (1990), Shea (1995), Parker (1999), Souleles (1999), Fuhrer and Rudebusch (2003), and Ahmad (2004).

14 Reported in the working paper version.
3.3 Parameter estimates and country-specific simulations

Table 2 summarizes the MCMC estimates of the structural parameters and their posterior distributions, obtained with the Metropolis-Hastings sampling algorithm. We find relevant heterogeneity across countries, in particular for the parameters indicating the fraction of rule-of-thumb households and habit persistence. The heterogeneity found with respect to the rule-of-thumb and the habit parameters emerges as the main cause of the differences that we get when the model is simulated employing the country-specific parameterization, while the other parameters show a lower cross-country variability.

Among the G7 countries, Italy shows the highest rate of habits (0.8), while Germany the lowest (0.6). The average habit persistence parameter is 0.7, a value strictly in line with the results obtained in previous empirical investigations.

The average fraction of Spenders for the G7 economies is 26%, well below the prior mean employed in the estimations, which has been chosen on the basis of our results from preliminary GMM estimates of reduced-form consumption functions and from other studies. Anyway, this value is broadly consistent with the outcomes of the analysis of Campbell and Mankiw (1991), who obtained a fraction of Spenders of approximately 35% for the U.S. and 20% for the U.K.. It is also marginally consistent with the results obtained by Banerjee and Batini (2003) who, employing the AIM solution procedure of Anderson and Moore, obtained a fraction of Spenders of nearly 26% for the US and of nearly 15% for the UK.

Interestingly, the fraction of rule-of-thumb households in Italy, Germany and Japan is relatively low (nearly 7% on average), while it is high in France (0.44), in the U.K. (0.42), in the U.S. (0.37) and in Canada (0.30). This result is surprising, since it requires explanations that are not in line with the standard view on the economic rationale behind rule-of-thumb consumption. In many studies the existence of Spenders is considered a proxy of the development and efficiency of the financial sector. As long as our estimates are reliable, since the highest fraction of Spenders is found for countries in which the financial markets are considered more developed and efficient, the standard
interpretation of rule-of-thumb consumption appears misleading. Under this perspective, differences are more likely to be related to institutional and cultural factors rather than to specific financial factors.\textsuperscript{15}

The estimates also show a considerable degree of Calvo price stickiness, which average estimate is 0.84, a value consistent with an average duration of the price contracts of approximately 6 quarters. We find a significant positive central bank’s short-term reaction to the current change in inflation and the output gap. Our estimation delivers plausible parameters for the long and short-run reaction function of the monetary authorities, and results are broadly in line with those discussed in Taylor (1993). The parameter for the policy reaction to inflation is rather stable across countries and in line with the prior assumptions. Some heterogeneity is found with respect to the policy elasticity to the output gap. The highest values are obtained for the U.S. and for Italy (nearly 0.2), while the lowest for Germany (0.11), Japan, France and the U.K.. In agreement with the large literature on estimated interest rate rules, we also find evidence of a substantial degree of interest rate smoothing, which in addition is also rather stable across countries.

The simulation of the DSGE model conducted employing the estimated structural parameters provides an appreciation of the degree of heterogeneity of the dynamic properties of the stylized economies. In particular, the simulations allow us to recognize the country specific efficacy of monetary policy and the degree of asymmetry of its effects. Figure 1 contains the impulse responses to a monetary policy shock, while Figure 2 the impulse responses to a technology shock.

In spite of the spenders, a positive monetary shock disturbance has the standard effects discussed above, i.e. it reduces both inflation and the real activity and increases the real interest rate. Hump-shaped responses are the usual result of habit persistence.

\textsuperscript{15} Despite different in many respects, Japan, Germany and Italy have some relevant similarities, as for the importance of the generational and family transfers and for the role and the features of the banking sector. Moreover, they show the highest saving rates among industrialized countries.
Concerning the country-specific inflation response to the monetary policy shock, the highest reaction is found for Japan, for which the half-life deviation from price stability is approximately four quarters, while the smaller is found to the U.S. which half life is nearly two quarters. The responses of output are even more differentiated among countries. A common feature is that the maximum effect on output of the monetary policy shock is reached after 2 quarters. The maximum responsiveness and duration of effects is found for the U.K., the minimum for the U.S.

About here figure 1.

The half life of the response is approximately 4 quarters for the U.S., 6 quarters for Italy, Germany and Japan, 7 for Canada and 8 quarters for the U.K. and France. In line with the theoretical predictions, with the exception of the U.S., the output sensitivity to monetary policy is thus stronger in those countries that show the highest fraction of rule-of-thumb consumers.

The technology shock $\varepsilon_i$ also has standard effects on the variables of the model. Inflation decreases at the impact following the marginal costs (Fig 8a-b). According to the monetary policy reaction rule, the nominal interest rate is decreased (Fig 8c), i.e. the policy accommodates the shock. The hump-shaped response of output, i.e. its deviation from the flexible price standard response, depends on the degree of inertia in policy.

About here figure 2.

As long as the nominal interest rate adjustment is smoothed by the monetary policy authorities, the real interest rate response may become positive, with counter-intuitive contractionary effects on output (via the IS equation). The estimated and simulated high degree of heterogeneity in policy response explains the heterogeneous impact and medium-term effects on output: they are in fact virtually zero at the impact for the majority of the countries considered in the analysis and negative for France and the U.K..

Even if emerging from a different perspective, this result is in line with the evidence produced by Gali (1999) on the possibility of “contractionary” supply shocks. If monetary policy does not fully accommodate the positive supply shock, the demand dynamics is unable to match the potential
output response, leading to the counter-cyclical conditional correlation between employment (hours) and productivity which has been addressed as the “productivity-employment puzzle.” The main difference here is that we do not consider this puzzle explicitly and, most importantly, that it emerges even considering a Taylor-like monetary rule instead of a money supply rule as in Gali (1999).

4. Conclusions

In this paper we have developed a simple New Keynesian DSGE model augmented with rule-of-thumb consumers and habit persistence in consumption. On the basis of these strong violations of the Hall’s benchmark consumption function, we have analyzed their implications for the stability properties of the model and, in particular, for the efficacy of the conduct of monetary policy. We have shown that the presence of rule-of-thumb consumers can potentially alter the conventional policy prescriptions.

We have formally discussed the relevance of the demand regimes in determining the properties of monetary policy and how demand regimes are affected by our assumptions on the consumers’ behavior. In particular, we have shown the importance of the sign and the size of the elasticity of the aggregate demand to the real interest rate, which determines, respectively, the demand regime and the monetary policy efficacy. Both elements (the demand regime and the monetary policy efficacy) individuate three possible policy regimes. More specifically, if the monetary policy is set according to a Taylor rule augmented with a stochastic disturbance, in the standard demand regime a negative interest rate disturbance always implies an increase in the output gap and inflation. By contrast, in the inverse regime two different situations can occur. For relative high values of monetary policy efficacy, a positive monetary disturbance will affect the economic outcomes as in the standard regime even if the elasticity of the aggregate demand with respect to the real interest rate is positive; conversely, for a relatively low monetary policy efficacy, the monetary shock has a deflationary effect as it will
decrease inflation and real output.

The rationale of these two different policy regimes in the case of a positive elasticity of the demand with respect to the real interest rate can be explained as follows: in the inverse regime a negative monetary shock initially shifts the aggregate demand backwards and, thus, it reduces real output and inflation. The reaction of the central bank to this change can imply either an increase or a reduction of the real interest rate.

In order to observe an inverse behavior of the economic variables with respect to a money disturbance, the inverse demand regime is thus a necessary but not sufficient condition because of the activity of the central bank. The reverse response is only observed for relative low value of the monetary policy efficacy, which makes the central bank reaction insufficient to reverse the direct effects of the disturbance.

The policy regime that will emerge depends on the monetary policy efficacy and the demand regime observed. Both depend on the elasticity of the aggregate demand, which in turn depends (non-linearly) on a large set of parameters. In the simplest case of no habit formation, the demand regimes are determined by a threshold value of the share of Spenders; the monetary policy efficacy is increasing in the share of Spenders in the standard regime whereas it is decreasing in the inverse regime. Thus, an inverse behavior of the economic variables to a monetary shock can be observed only for a very high share of Spenders, i.e. for values consistent with values supporting the inversion of the demand regime. The strong parameters non-linearity obtained when habits are introduced makes the derivation of general results a difficult task. Numerical simulations show that, *ceteris paribus*, the threshold value of the fraction of Spenders needed to obtain the monetary multiplier sign inversion tends to increase in the parameter defining habit persistence, hence, by introducing habits, the probability of observing a demand regime shift decreases.

The presence of relevant non-linearities requires a careful calibration of the model for running policy experiments and for the understanding the policy transmission dynamics. The empirical relevance of our hypotheses has thus been evaluated estimating the structural parameters of the
DSGE model for the seven most industrialized economies, and then employing these structural estimates for obtaining country-specific simulations of the dynamics of the stylized economies.

Our simulations have evidenced the efficacy of the monetary policy in stabilizing the business cycle in all the countries considered. They have also highlighted the presence of relevant international asymmetries in the monetary transmission mechanisms. The presence of asymmetries in the monetary transmission channels stimulates a serious reconsideration of the policy prescriptions, in particular of those that have been obtained without taking into account that the differences among economies may result decisive in the determination of the effects of the policy. Anyway, despite the heterogeneous sensitivity to shocks, the dynamical properties of the models resulted qualitatively in line with those predicted by the conventional New Keynesian DSGE model. In particular, the estimated structural parameters rule out the possibility of demand regime inversions due to the presence of rule-of-thumb consumers. Notwithstanding the fraction of Spenders is relevant in many countries (0.26 on average), in none of them this fraction is high enough to generate the regime inversion. A further interesting result is that, despite the model is theoretically able to generate the so-called “price puzzle” for habits and rule-of-thumb parameters values that are not prohibitively high, the estimation has generated a parameterization that is not consistent with this result.

The main outcome of our analysis is thus that, even if the theoretical implications of the presence of rule-of-thumb consumers and of persistence in consumption habits are potentially strong, their empirical evaluation suggests that the model properties remain qualitatively unchanged. The estimated degree of habit persistence and fraction of Non Ricardian consumers, even if dimensionally relevant on average, are not able to overturn the qualitative properties of the standard New Keynesian sticky price model. However, the analysis has also shown that these modifications should be taken in serious consideration, since their presence affects the relative efficacy of the monetary policy and can influence the symmetry of its effects.
Appendix A – Determinacy

Determinacy is studied by augmenting the log-linearized dynamic system with a simple Taylor feedback rule:\(^\text{16}\)

\[
\begin{bmatrix}
1 & \Omega \\
0 & \beta
\end{bmatrix}
E_t
\begin{bmatrix}
y^t_{t+1} \\
\pi^t_{t+1}
\end{bmatrix}
=
\begin{bmatrix}
1+\Omega a_2 & \Omega a_t \\
-k & 1
\end{bmatrix}
\begin{bmatrix}
y^t \\
\pi^t
\end{bmatrix}
\]

Stability depends on the eigen-structure of the following matrix:

\[
M = \begin{bmatrix}
1 & \Omega \\
0 & \beta
\end{bmatrix}^{-1}
\begin{bmatrix}
1+\Omega a_2 & \Omega a_t \\
-k & 1
\end{bmatrix}
= \begin{bmatrix}
1+\Omega \left( a_2 + \frac{k}{\beta} \right) & \Omega \left( a_t - \frac{1}{\beta} \right) \\
-k \frac{1}{\beta} & \frac{1}{\beta}
\end{bmatrix}
\]

By indicating with \( D(.) \) and \( T(.) \) the determinant and trace operators, we have:

\[
\begin{align*}
D(M) &= \beta^{-1} + \Omega (a_2 + k a_t) \beta^{-1} \\
T(M) &= 1 + a_2 \Omega + (1 + k \Omega) \beta^{-1}
\end{align*}
\]

The eigen-structure of matrix \( M \) is studied by following Woodford (2003: Appendices to Chapter 4). Since the analysis of the standard one does not differs from Woodford (2003), we only consider the liquidity-constrained regime. In this regime, determinacy requires either: \( i) \) \( D(M) > 1 \), i.e. \( a_t < \left[ (1 - \beta) \overline{\Omega}^{-1} - a_2 \right] k^{-1} \), \( D(M) \pm T(M) + 1 > 0 \) or \( ii) \) \( D(M_1) \pm T(M_1) + 1 < 0 \).

Being:

\[
\begin{align*}
D(M) + T(M) + 1 &= \left[ 2 (1 + \beta) - \overline{\Omega} \left[ (1 + \beta) a_2 + (1 + a_t) k \right] \right] \beta^{-1} \\
D(M) - T(M) + 1 &= -\overline{\Omega} \left[ (1 - \beta) a_2 + k (a_t - 1) \right] \beta^{-1}
\end{align*}
\]

from equations (A.4) and (A.5) we derive the reported in section 2.3.

\(^{16}\) In order to investigate the stability properties we do not need to look at the stochastic part and constants that thus are omitted for the sake of brevity. We assume stationary disturbance processes.
Appendix B – Monetary policy transmission (policy regimes)

By simple derivation we obtain

\[ \frac{\partial y_t}{\partial \varepsilon_t} = \Omega \frac{\partial i_t}{\partial \varepsilon_t}; \quad \frac{\partial \pi_t}{\partial \varepsilon_t} = \kappa \frac{\partial y_t}{\partial \varepsilon_t}; \quad \frac{\partial i_t}{\partial \varepsilon_t} = a_1 \frac{\partial \pi_t}{\partial \varepsilon_t} + a_2 \frac{\partial y_t}{\partial \varepsilon_t} + 1, \]

which combined give

\[ \frac{\partial y_t}{\partial \varepsilon_t} = \Omega \left( a_1 \frac{\partial y_t}{\partial \varepsilon_t} + a_2 \frac{\partial y_t}{\partial \varepsilon_t} + 1 \right), \quad \frac{\partial y_t}{\partial \varepsilon_t} = \frac{\Omega}{1 - \Omega(a_1 \kappa + a_2)}; \quad \frac{\partial \pi_t}{\partial \varepsilon_t} = \frac{\Omega \kappa}{1 - \Omega(a_1 \kappa + a_2)} \]

and

\[ \frac{\partial i_t}{\partial \varepsilon_t} = \frac{1}{1 - \Omega(a_1 \kappa + a_2)}, \]

from which the discussion in the main text can be easily derived.

References


Di Bartolomeo, G. and L. Rossi (2005), Heterogeneous Consumers, Demand Regimes, Monetary Policy Efficacy and Determinacy, *EACB Research Paper*.


Ireland, P.N. (1999), A Method for Taking Models to the Data, Boston College, mimeo.


Box 1 – The theoretical model (A summary).

\[ y_t = \Omega_1 E_t y_{t+1} + \Omega_2 y_{t-1} - \Omega_3 \left( i_t - E_t \pi_t - \mu_{i_t}^{IS} + E_t \mu_{i_t}^{IS} \right) + \Omega_4 E_t \Delta a_{t+1} \]

\[ \pi_t = \beta E_t \pi_t + \tau m c_t \]

\[ i_t = \rho_i a_{t-1} + \left( 1 - \rho_i \right) \left[ \pi_t^* + \psi_\pi \left( \pi_t - \pi_t^* \right) + \psi_x x_t \right] + \varepsilon_t \]

\[ m c_t = \frac{1 + \nu \left( 1 - \sigma \right)}{2} y_t + \frac{\sigma}{2} y_{t-1} - \left( 1 + \nu \right) a_t + \varepsilon_t^{op} \]

\[ x_t = y_t - y_t' \]

\[ y_t' = \frac{(1 + \nu) \left( 1 - \sigma \right)}{1 + \nu \left( 1 - \sigma \right)} a_t + \frac{\sigma}{1 + \nu \left( 1 - \sigma \right)} y_t'_{t-1} \]

\[ \pi_t^* = \rho_x \pi_{t-1}^* + \varepsilon_t^{\pi} \]

\[ a_t = \rho_a a_{t-1} + \varepsilon_t^{a} \]

\[ \mu_t^{IS} = \rho_{i_t} \mu_{i_t}^{IS} + \varepsilon_t^{IS} \]

### Table 1. Prior distributions for the structural parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Prior shape</th>
<th>Prior mean</th>
<th>Prior S.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>sigma_e_a</td>
<td>Structural technology shock</td>
<td>inv_gamma</td>
<td>0.090</td>
<td>2</td>
</tr>
<tr>
<td>sigma_e_I</td>
<td>Structural technology shock</td>
<td>inv_gamma</td>
<td>0.220</td>
<td>2</td>
</tr>
<tr>
<td>sigma_e_pi</td>
<td>Structural technology shock</td>
<td>inv_gamma</td>
<td>0.010</td>
<td>2</td>
</tr>
<tr>
<td>sigma_e_i</td>
<td>Structural technology shock</td>
<td>inv_gamma</td>
<td>0.012</td>
<td>2</td>
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<tr>
<td>sigma_e_dP</td>
<td>Structural technology shock</td>
<td>inv_gamma</td>
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<td>2</td>
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<tr>
<td>rho_a</td>
<td>Persistence parameter for tech. shock</td>
<td>beta</td>
<td>0.850</td>
<td>0.10</td>
</tr>
<tr>
<td>rho_IS</td>
<td>Persistence parameter for tech. shock</td>
<td>beta</td>
<td>0.850</td>
<td>0.10</td>
</tr>
<tr>
<td>rho_pi</td>
<td>Persistence parameter for tech. shock</td>
<td>beta</td>
<td>0.850</td>
<td>0.10</td>
</tr>
<tr>
<td>rho_i</td>
<td>Smoothness parameter for nominal interest</td>
<td>beta</td>
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<td>0.10</td>
</tr>
<tr>
<td>beta</td>
<td>Discount factor</td>
<td>-</td>
<td>0.995</td>
<td>0</td>
</tr>
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<td>eta</td>
<td>Elasticity of substitution among intermediate goods</td>
<td>normal</td>
<td>6.000</td>
<td>0.30</td>
</tr>
<tr>
<td>k</td>
<td>Labor disutility</td>
<td>normal</td>
<td>3.000</td>
<td>0.15</td>
</tr>
<tr>
<td>psi_pi</td>
<td>Taylor rule parameter on inflation</td>
<td>normal</td>
<td>1.500</td>
<td>0.15</td>
</tr>
<tr>
<td>psi_x</td>
<td>Taylor rule parameter on output gap</td>
<td>normal</td>
<td>0.125</td>
<td>0.05</td>
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<td>phi</td>
<td>Calvo parameter</td>
<td>beta</td>
<td>0.750</td>
<td>0.10</td>
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<tr>
<td>gamma</td>
<td>Habits persistence parameter</td>
<td>beta</td>
<td>0.700</td>
<td>0.10</td>
</tr>
<tr>
<td>lambda</td>
<td>Fraction of rule of thumb consumers</td>
<td>beta</td>
<td>0.500</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Note: for the inverted gamma distribution the degrees of freedom are indicated
Table 2. MCMC estimates of the structural parameters. G7 countries

<table>
<thead>
<tr>
<th>Parameter</th>
<th>USA</th>
<th>JAP</th>
<th>GER</th>
<th>FRA</th>
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</thead>
<tbody>
<tr>
<td>sigma_e_a</td>
<td>0.048</td>
<td>0.048</td>
<td>0.048</td>
<td>0.048</td>
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<tr>
<td>sigma_e_I</td>
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<td>0.113</td>
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<td>sigma_e_pi</td>
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<td>0.005</td>
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<tr>
<td>sigma_e_i</td>
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<td>0.175</td>
<td>0.241</td>
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<td>rho_a</td>
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<td>0.735</td>
<td>0.767</td>
<td>0.828</td>
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<tr>
<td>rho_I</td>
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<td>0.946</td>
<td>0.949</td>
<td>0.935</td>
</tr>
<tr>
<td>rho_pi</td>
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<td>0.744</td>
<td>0.763</td>
<td>0.970</td>
</tr>
<tr>
<td>rho_i</td>
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<td>0.801</td>
<td>0.803</td>
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<td>beta</td>
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<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
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<td>5.810</td>
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<td>3.143</td>
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Figure A7. Impulse responses to a monetary policy shock, M-H MCMC estimates

a) Inflation

b) Marginal costs

c) Nominal interest rate

d) Output

Computations obtained with Dynare for Matlab.
Figure A8. Impulse responses to a technology shock, M-H MCMC estimates

- **a)** Inflation
  - CAN
  - ITA
  - USA
  - GER
  - UK

- **b)** Marginal cost
  - CAN
  - ITA
  - USA
  - GER
  - UK

- **c)** Nominal interest rate
  - CAN
  - ITA
  - USA
  - GER
  - UK

- **d)** Output
  - CAN
  - ITA
  - USA
  - GER
  - UK

Computations obtained with Dynare for Matlab.