Labour market imperfections, “divine coincidence” and volatility of employment and inflation

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PROVISIONAL DRAFT
Abstract

The dynamic general equilibrium model with hiring costs presented in this paper delivers involuntary unemployment in the steady state as well as involuntary fluctuations in unemployment. The existence of hiring friction introduces externalities that, in turn entail the breakdown of the “divine coincidence” without assuming real wage rigidity. We are able to show that our model with labour market imperfections outperforms the standard NK model as for the persistence of responses to monetary shocks. We also attempt an analysis of the volatility of two economies, differing in their “degrees of imperfection”. It turns out that “rigid” economies exhibit less unemployment volatility and more inflation volatility than “flexible” economies.

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1 Introduction and motivation

1.1 The shortcomings of the NK model

Since the late Nineties a standard New Keynesian (NK) dynamic general equilibrium model has emerged which is widely used as a work-horse for monetary policy. Such a model is built on microfoundations coming from the Real Business Cycle (RBC) i.e. intertemporal optimisation of infinitely lived, fully rational, consumers and firms. The NK model departs from the RBC in assuming imperfect competition in the products market and staggered prices à la Calvo (1983). As a result of this blend of RBC and Keynesian ingredients a NK Phillips curve is derived which implies that monetary policy can have relevant effects on real output, something the RBC model alone cannot deliver\(^1\).

However, it is now commonly acknowledged that the standard NK model has three main shortcomings: 1) there is no involuntary unemployment, because of the hypothesis of a Walrasian labour market; 2) there is no trade-off between inflation and output gap stabilisation; 3) contrary to empirical evidence, in the model the inflation response to shocks is greater than the output response, whilst output fluctuations cannot be as persistent as they appear to be in the real world.

The absence of involuntary unemployment is a serious shortcoming for a model labelled as “Keynesian”, however abridged or reformed. In the standard NK model output fluctuations imply that people vary the hours they work (variation of the intensive margin) but the number of people employed never changes (that is, there is no variation of the extensive margin). Such an un-Keynesian feature of the NK model is at odds with empirical evidence, which does show changes in the number of people working whilst does not show a labour supply as wage elastic as needed for the adjustment to take place along the intensive margin alone (Trigari, 2005; Faia, 2007; Ravenna, Walsh, 2007).

The absence of a trade-off between inflation and output stabilisation in the standard NK model has been christened “divine coincidence” (Blanchard, Galí, 2005). The divine coincidence “is tightly linked to a specific property of the standard NK model, namely the fact that the gap between the natural level of output and the efficient (first best) level of output is constant and invariant to shocks” (Blanchard, Galí, 2005, p. 2). Such a feature of the standard NK model entails that stabilising the actual output gap (i.e. the difference between actual and “natural” output) is equivalent to stabilising the welfare relevant output gap (i.e. the difference between actual output and first best output). As stabilising inflation also stabilises the actual output gap, the standard NK model implies that stabilising inflation brings about stabilisation of the welfare relevant output gap: a divine coincidence indeed. A divine coincidence that makes inflation targeting surrounded by a halo of optimality\(^2\).

As for the inability at delivering enough persistence of output fluctuations after a nominal shock, it may be argued that the presence of nominal rigidities is not able to overcome the RBC feature of the model, in which forward looking workers and firms are able to rapidly adjust their hiring and working decisions in a perfectly competitive labour market. In a Walrasian labour market, fluctuations in employment levels are interpreted as the outcome of voluntary choices and must be accompanied by real wage changes: a temporary increase in the current wage leads workers to offer more labour services in the current period, in exchange for more leisure in the future. However, a smoother correlation between wages and employment is frequently observed, and this evidence is

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\(^1\)This earlier literature, described by Goodfriend and King (1997), has often been labelled as “New Neoclassical Synthesis”.

\(^2\)“The present theory implies not only that price stability should matter in addition to stability of the output gap, but also that, at least under certain circumstances, inflation stabilization eliminates any need for furthur concerner with the level of real activity. This is because [...] the time-varying efficient level of output is the same (up to a constant, which does not affect the basic point) as the level of output that eliminates any incentive for firms on average to either raise or lower their prices”. (Woodford, 2003, p. 13)
at variance with the theoretical RBC predictions, unless the (real) wage elasticity of labour supply is implausibly high\(^3\).

With a Walrasian labour market, it is difficult to offer some plausible rationales for the insensitive reaction of marginal costs to demand shocks. The missing explanation for acyclical real wage patterns is at the root of an intrinsic inability of the standard NK model to reproduce the low sensitivity of real marginal costs to output changes and to replicate the sluggishness in price setting behavior. Only by assuming a high degree of nominal inertia - which prevents firms from full price adjustments - one may preserve the hypothesis of a sensitive marginal cost and still obtain the stickiness in price behaviour observed in reality. However, microeconomic data on price setting show that the majority of firms resets their prices more frequently than once a year (see for instance Blinder et al., 1998, and Carlton, 1986). The effective role of nominal frictions has been raised by Chari, Kehoe and McGrattan (2000), by showing that, for a wide range of parameter values of a specified model with a non-competitive product market, the hypothesis of staggering alone does not succeed in explaining the size and persistence of observed cyclical fluctuations.

### 1.2 Labour market imperfections and real wage rigidities

Many attempts have recently been made at overcoming the above mentioned shortcomings of the standard NK model. Not surprisingly, most of these attempts point to some sort of labour market imperfection\(^4\). An early study in this direction (Jeanne, 1998) showed that the introduction of significant real rigidities due to a non-competitive labour market and to an a-cyclical wage dynamics, strengthens nominal rigidities and is compatible with large-scale cyclical fluctuations which persist over time.

In a few recent papers (e.g. Christoffel Linzert, 2005, Trigari, 2005 and Walsh, 2005), search frictions are introduced alongside a Mortensen, Pissarides (1994, 1999) matching function\(^5\). In this framework workers and firms bargain over wages and share the positive rents arising from a successful match. However, this rule makes the wage proportional to productivity changes or to changes in labour market tightness, which means that labour compensations absorb and filter exogenous shocks. Thus, in case of a positive shock, little space is left for the opening of new vacancies, while in case of an adverse shock, the low recruiting effort of employees is still unexplained. This means that the matching model does not account for the variability of vacancies and does not reproduce the employment fluctuations observed in reality\(^6\).

It is only under some stickiness in the real wage, as that obtained by Hall (2005) with the assumption of a wage norm, that the Mortensen and Pissarides approach gains more empirical relevance. However, Hall explicitly admits that he does not “venture into the territory of explaining why the economy appears to choose sticky wages from the wide variety of alternative equilibrium wage patterns” (Hall, 2005, 51). In fact the studies mentioned above combine searching frictions and real wage rigidity in order to obtain a model economy where plausible output and inflation dynamics are obtained.

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\(^3\)The empirical evidence reveals a low elasticity of employment to the real wage. See, for instance, Pencavel (1986).

\(^4\)Alongside these attempts another strand of research grew aimed at introducing additional rationales for nominal rigidities: from the sticky information approach, developed by Mankiw and Reis (2002), to the staggered nominal wage contracts approach, proposed by Christiano, Eichenbaum and Evans (2005), from the rule of thumb behavior in price or wage setting, advanced by Gal and Gertler (1999) and Rabanal (2001), to the lagged indexation assumption advanced by Smet and Wouters (2003) and Christiano, Eichenbaum and Evans (2005).

\(^5\)Attempts at introducing efficiency wages in a dynamic general equilibrium model have been made by Felices (2002), Alexopoulos (2004), Danthine and Kurmann (2004).

\(^6\)For instance, in the U.S., as argued in Shimer (2005), the standard deviation of the vacancy-unemployment ratio is almost 20 times as large as the standard deviation of productivity, while the search model predicts the same volatility. Analogous evidence is observed by Hall (2005) in case of demand shocks, thus proving that “recessions are times when the labor markets of all industries slacken - not times when workers moves from industry with slack markets to other with tight markets” (Hall, 2005, p. 52).
Blanchard, Galí (2005) bypass the labour market imperfection issue by assuming real wage stickiness straight away. They are able to show that when real wage stickiness alone is introduced in an otherwise standard NK model the divine coincidence disappears, as “the gap between natural and efficient output is no longer constant, and is now affected by shocks” (p. 3). As a consequence, stabilising inflation is no longer equivalent to stabilising the welfare relevant output gap and inflation targeting is no longer optimal: policy makers are faced with a trade-off between stabilising inflation and stabilising the welfare relevant output gap. In a more recent paper, Blanchard, Galí (2006) introduced labour market frictions alongside real wage stickiness, obtaining basically the same results as in their 2005 paper.

1.3 The present paper

In a previous paper (Abbritti, Boitani, Damiani, 2006) we proved that by introducing labour market imperfections à la Howitt in a New Keynesian model it is possible to have, at once, involuntary unemployment, the end of the divine coincidence and persistent output and employment fluctuations with no need for real wage rigidity. The present paper is a refinement and an extension of the previous one. The refinement consists in (a) the adoption of a more standard households’ utility function; (b) the adoption of Blanchard Galí (2006) specification of the hiring cost function; and (3) the use of constrained efficient equilibrium as a steady state target that the benevolent social planner is able to reach by means of lump-sum subsidies to unemployed workers.

By so doing we make it easier to compare our results with those obtained in the recent literature. At the same time we preserve the tractability of the model, which we regard as an important “plus”. This is achieved by not going all the way down to allow both intensive and extensive margin adjustments or to introduce a full-fledged search and matching model of the labour market. We will analyse how inflation and employment fluctuations are influenced by labour market institutions by taking explicitly into account their interplay and without ignoring that different labour market rigidities indicators are usually linked in a predictable way. For instance, a lower separation rate, and therefore a higher labour protection, is usually associated to higher values of all the other factors that cause higher unemployment rates and lower job finding rates.

This refined model maintains the capability of delivering involuntary unemployment as a steady state equilibrium and the breakdown of divine coincidence, without an exogenous real wage stickiness. The existence of a dynamic externality - due to the effects of hiring decisions on hiring costs faced by firms - makes the dynamic behaviour of a decentralised economy different from that of the constrained efficient economy and leads the divine coincidence to vanish. After allowing for staggered pricing à la Calvo, we show that - due to the absence of divine coincidence - neither is pure inflation targeting able to stabilise employment fluctuations nor is pure employment targeting able to stabilise inflation, as it would be the case in a standard NK model.

As many other models with labour market imperfections, our model outperforms the standard NK model as for the persistence of responses to monetary shocks. In order to test the ability of our model to fit some real-world feature of economic dynamics, we identify a measure of economy-wide labour market rigidity, based on the unemployment rate and on the job finding rate (but not on real wage stickiness). This allows us to simulate the inflation and output volatility of two economies differing in their degree of labour market rigidity. We find that a “rigid” economy (according to our measure) displays higher unemployment volatility and lower inflation volatility than a “flexible” economy. A result that is broadly consistent with the findings of Giannone, Reichlin (2006) referred to the flexible US and the rigid Euro area.

The rest of the paper is organised as follows. In section 2 the most controversial aspects of the standard NK model are sketched out for future reference. Section 3 is devoted to the building blocks of the model with hiring costs, i.e. the (by now standard) derivation of a new IS curve from utility maximisation and that of hiring, pricing and employment decisions. In section 4 the decentralised
and the constrained efficient equilibria of this economy are derived and compared, stressing the role played by labour market externalities in the dynamics of a decentralised economy. Section 5 deals with staggered prices and shows the breakdown of the divine coincidence. After presenting the reduced model in section 6, and discussing the parameters used to calibrate the model (section 7), we compare the dynamics of our model with hiring frictions and that of a standard NK model in section 8. Section 9 presents our analysis of the relationship between labour market rigidity and the volatility of unemployment and inflation. Section 10 briefly concludes.

2 At the heart of divine coincidence

The standard NK models integrate imperfect competition and nominal rigidities into a dynamic general equilibrium framework largely associated with the RBC paradigm (Galí 2002). In particular, the NK model inherits from the RBC literature a neoclassical labour market. This fact leads, in our view, to some of the weaknesses of this model.

In a standard NK model, the period utility function depends on consumption $C_t$ and on hours worked $h_t$:

$$U(C_t, h_t) = C_t^{1-\sigma} - \frac{\xi h_t^{1+\nu}}{1+\nu}$$ (1)

Utility maximization leads to a standard labour supply equation where the real wage equals the marginal rate of substitution:

$$\frac{W_t}{P_t} = \xi \frac{h_t^\nu}{C_t^\sigma}.$$ On the supply side, firms choose prices taking into consideration the marginal cost’s dynamics, which simply reflects movements in real wages and productivity:

$$MC_t = \frac{W_t}{A_tP_t} = \xi \frac{h_t^\nu}{A_tC_t^\sigma}.$$ (2)

This specification of the labour market is at the heart of some of the criticism around the NK model. As the labour supply turns out to be binding in equilibrium, the model is unable to explain involuntary unemployment.

Moreover, in the standard NK model a meaningful policy trade-off between output and inflation stabilisation is absent. To see this, consider the New Keynesian Phillips Curve:

$$\pi_t = \beta E_{t+1} \pi_t + \lambda z_t$$ (3)

where $\pi_t$ is inflation and $z_t$ is the output gap. Iterating forward, one can express the current inflation rate in terms of current and future output gaps:

$$\pi_t = \lambda \sum_{s=0}^{\infty} \beta^s E_t \{z_{t+s}\}$$ (4)

Using (3), it is easy to show that a pure inflation targeting strategy, i.e. a strategy where $\pi_t = 0$ at all $t$, completely stabilises the output gap, i.e. $z_t = 0$ at all $t$. Viceversa, a strategy that stabilises the output gap in each period, setting $z_t = 0$ at all $t$, completely stabilises inflation ($\pi_t = 0$ at all $t$). Hence the monetary authority does not face a policy trade-off between output and inflation stabilisation: this is the essence of “divine coincidence” (Blanchard Galí, 2005). Such a divine coincidence is seen as unsatisfactory by many researcher and central bankers. In the following sections we shall argue that, even without imposing explicitly some form of real wage rigidity, the divine coincidence disappears as soon as a more realistic structure for the labour market is introduced.
3 Flexible price equilibrium

This section presents a simple dynamic stochastic general equilibrium model with labour and product market imperfections. There are three groups of agents: households, firms and a monetary authority. Households maximise lifetime utility derived from consumption of a composite good and from leisure. Monopolistically competitive firms maximise profits by choosing prices and employment levels, under the constraint of hiring costs and facing an exogenous separation rate. Workers and employers bargain over wages: the two parties share the positive surplus arising from a successful hiring. Under fully flexible prices and wages the central bank’s only role is fixing the inflation rate.

3.1 Households

The representative household is thought of as a continuum of members with names on the unit interval. In equilibrium, some individuals will be employed and others will not be; to avoid distributional issues, we follow most of the literature in assuming that households perfectly insure each other against fluctuations in consumption.

Lifetime utility depends on the consumption of the family and on the household’s disutility of work:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \xi N_t \left( \frac{\bar{h}^{1+\upsilon}}{1+\upsilon} \right) \right\}$$

where \(\beta\) is the subjective discount factor and \(\sigma\) is the constant degree of relative risk aversion and

\[\chi = \xi \left( \frac{k^{1+\upsilon}}{1+\upsilon} \right)\]

Notice that the disutility of labour for the household is the aggregate of the individuals’ disutility of labour. Empirical evidence suggests that most of the labour adjustment takes place along the extensive margin. Accordingly and for simplicity, we assume that each individual works a fixed number of hours \(h_t = \bar{h}\). Hence we rule out by assumption all adjustments along the intensive margin. The utility function is thus linear in the number of the employed people.

Households own all firms in the economy and face, in each period, the following budget constraint:

$$C_t + \frac{B_t}{P_t (1+i_t)} = D_t + \frac{B_{t-1}}{P_t}$$

where \(C_t\) is a standard Dixit-Stiglitz consumption bundle with elasticity of substitution \(\epsilon\), \(P_t\) is the aggregate price level, \((1+i_t)\) is the gross nominal interest rate of one-period bond and \(D_t\) is the per capita family income in period \(t\), which is the sum of the wage income earned by employed family members \((W_tN_t)\) and the family share of aggregate profits from firms, net of government lump-sum taxes.

Solving the intertemporal optimisation problem one gets the following first order condition:

$$C_t^{1-\sigma} = \beta(1+i_t)E_t \left( C_{t+1}^{1-\sigma} \frac{P_t}{P_{t+1}} \right)$$

Log-linearising equation (6) around the steady state, one gets the new IS curve (McCallum and Nelson, 1999):

$$\dot{c}_t = E_t \dot{c}_{t+1} - \frac{1}{\sigma} (\dot{i}_t - E_t \dot{\pi}_{t+1})$$
where variables with a hat denote log-deviations from steady state and $\pi_t = \log \frac{P_t}{P_{t-1}}$ is the inflation rate at time $t$.

3.2 Firms and the labour market

3.2.1 Hiring decisions

In the economy there is a continuum of firms, each producing a differentiated good with an identical technology:

$$Y_t^i = A_t N_t^i$$

where productivity $A_t$ follows an AR(1) process.

In such a model, employment dynamics can be defined by assuming an average separation rate equal to $\delta$, where $\delta \in (0, 1)$, and on the basis of an optimum hiring rate equal to $H_t$, endogenously determined as the outcome of optimal choices by individual firms. The separation rate $\delta$, which is a measure of the probability of job termination, is simply considered as an exogenous parameter in some of the literature, even if some other studies have tried to provide an endogenous determination (see, e.g. Trigari, 2005; Walsh, 2005). The parameter $\delta$ can be interpreted as the inverse of the degree of labour protection.

The evolution of employment at firm $i$ is determined by the following:

$$N_t^i = (1 - \delta) N_{t-1}^i + H_t^i$$

At the aggregate level, employment $N_t \equiv \int_0^1 N_t^i di$ evolves according to the following:

$$N_t = (1 - \delta) N_{t-1} + H_t$$

where $H_t \equiv \int_0^1 H_t^i di$ denotes the aggregate hiring level.

We denote by $U_t$ the pool of jobless individuals who are available for new jobs. Since we make assumptions below that ensure full participation and the labour force is normalised to 1, $U_t$ is defined as follows:

$$U_t = 1 - (1 - \delta) N_{t-1}$$

After hiring decisions are undertaken, unemployment is defined as $u_t = 1 - N_t$.

The optimal hiring decisions are made under the hypothesis, suggested in Howitt (1988), that firms face a cost of searching and recruiting new workers. Specifically, we assume, as in Blanchard-Galì (2006), that hiring costs for firm $i$ are given as follows:

$$G_t H_t^i$$

where unit hiring costs are an increasing function of the labour market tightness index $x_t = H_t U_t$:

$$G_t = A_t B (x_t)^\alpha = A_t B \left( \frac{H_t}{U_t} \right)^\alpha$$

where the elasticity of the hiring cost function $\alpha > 0$ and $B$ is a scaling parameter that may be influenced by the policy maker.

The relevance of $G_t$ in our model economy is strictly related to the extensive margin hypothesis: each firm may adjust its optimal amount of labour by recruiting additional workers and thus paying the hiring cost; the relevance of hiring costs emerges even in more general models, where extensive margin adjustments are accompanied by intensive margin adjustments, provided the first kind of adjustment does not play a trivial role. Firms may bear advertising, screening, and training costs.

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7We assume that wages are set such that any individual is willing to work.
and may incur in firing costs when protection legislation imposes legal restrictions. In this context, search and labour market frictions, explored along the lines suggested by Mortensen and Pissarides (1994), are accompanied by turnover costs. Furthermore, the marginal cost of hiring is increasing in the aggregate level of hiring $H_t$; this captures the idea that a high rate of hiring may force firms to increase their search intensity. That means that with an increase in employment due to hiring ($H_t$) a “congestion” effect occurs: the recruitment process becomes more difficult and the matching less favourable. Viceversa, with an increase in $U_t$, it is easier for the firm to recruit workers, and the matching between the skills required by the firm and those offered by the available work-force improves.

### 3.2.2 Price and wage determination

Under flexible prices, the optimal price setting rule for firm $i$ is $^8$:

$$\frac{p^i_t}{P^i_t} = \mu \{ MC_t \}$$  \hspace{1cm} (13)

where the term $\mu = \frac{1}{c-1}$ is the mark up.

With flexible prices, in a symmetric equilibrium, all firms will charge the same price ($p^i_t = P^i_t$). This implies that the real marginal cost will be constant and equal to the inverse of the markup:

$$MC_t = \frac{1}{\mu}$$  \hspace{1cm} (14)

The current expected value of the marginal cost $MC_t$ is affected by the presence of hiring costs as follows:

$$MC_t = \frac{W_t}{A_t P_t} + \left\{ B x^\sigma_t \beta (1 - \delta) E_t \left[ \left( \frac{C^\sigma_t}{C^\sigma_{t+1}} \right) B x^\sigma_{t+1} \right] \right\}$$  \hspace{1cm} (15)

By inspection of (15) one can see that hiring new workers at time $t$ has two effects: i) it increases the recruitment costs at time $t$ - an effect represented by the term $B x^\sigma_t$; ii) it reduces the costs of hiring new workers in period $t + 1$, since higher levels of recruiting efforts undertaken in the first period decrease the needs for firms to hire in the following period. The second effect is captured by the last term in (15). In this model the presence of hiring costs creates a wedge between the real wage and the marginal cost relevant for the firm, which turns out to be crucial in the explanation of inflation dynamics. Such a wedge leads the cyclical behaviour of marginal costs in a model with labour market imperfections to substantially deviate from that of real wages (compare 15 with 2).

As Krause and Lubik (2005, p. 11) notice, “hiring frictions generate a surplus for existing matches which give rise to long-term employment relationships. These, in turn, reduce the allocative role of current real wages. As a consequence, the effective real marginal cost can change even if the wage does not change”.

The presence of search frictions creates a positive rent for existing employment relationships. Following much of the literature, we assume wages are bargained to split this rent between the firm and the employee, according to their respective bargaining power (Nash bargaining).

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$^8$ Firms, in each period, choose the price of its product to maximize:

$$\text{Max}_{p^i_t,h^j_t} \sum_{s=0}^{\infty} Q_{t+s} \{ p^i_{t+s} Y^j_{t+s} - P^i_{t+s} G_{t+s} h^j_{t+s} - W_{t+s} N^j_{t+s} \}$$

subject to the demand function

$$Y^j_t = \left( \frac{p^j_t}{P^j_t} \right)^{-\sigma} [C_t + G_t H_t]$$

and to the production function and the employment evolution equation. $Q_{t+s} = \beta \frac{C^\sigma_t}{C^\sigma_{t+s}} \frac{p^i_t}{P^i_{t+s}}$ is the relevant stochastic discount factor for nominal payoffs.
Let $\eta$ denote the relative weight of workers in the Nash bargaining. It can be shown that the Nash bargained wage (normalised by productivity) is given by:

$$W^\text{Nash}_{t} = \frac{\chi}{A_t C_t^{-\sigma}} + \eta \left\{ B x_t^\rho - \beta (1 - \delta) E_t \left[ \left( \frac{A_{t+1}}{A_t} \frac{C_{t+1}^\sigma}{C_t^\sigma} B x_{t+1}^\alpha \right) (1 - x_{t+1}) \right] \right\} \quad (16)$$

Intuitively, the Nash wage depends on the reservation wage (here given by the marginal rate of substitution between leisure and consumption, $\chi_{A_tC}$) plus a volatile “wage premium”, which depends on the size of the rents for existing employment relationships (the term in curled brackets) and on the workers’ relative share of the surplus, $\eta$. The lower is $\eta$ the less influenced is the Nash wage by the possibly volatile components in (16). The size of the rents is determined by the hiring cost component of the marginal costs in (15), and by taking into account that the probability not to find a job in the next period $(1 - x_{t+1})$ exerts a disciplinary influence on current bargaining; because it reduces the future workers’ share of the matching surplus and thus exerts a disciplinary influence on today’s current bargaining. By inspection of (15) and (16) one can thus easily verify that hiring frictions cause a double dynamic effect: the next period tightness of the labour market causes a larger increase in hirings today (the direct effect) and, additionally, it influences the wage setting (the indirect effect).

4 The natural and constrained efficient equilibria

Substituting the wage schedule (16) in the expression for the marginal costs, we get the natural equilibrium under Nash bargaining:

$$\frac{\chi}{A_t C_t^{-\sigma}} = \frac{1}{\mu} - (1 + \eta) B x_t^\rho + \beta (1 - \delta) E_t \left\{ B x_{t+1}^\alpha \right\} \quad (17)$$

The left hand side represents the marginal rate of substitution between consumption and labour; the right hand side the corresponding marginal private rate of transformation (both normalised by productivity). In an equilibrium with flexible prices and Nash bargained wages, employment is invariant to productivity only if the utility function is log in consumption, i.e. $\sigma = 1$.

It is also possible to solve the problem of a benevolent social planner that maximises the welfare of the representative household in an economy with the technological constraints and labour market frictions described so far. Notice that the social planner internalises the effects of variations in employment on hiring costs. Hence the equilibrium one finds is a constrained efficient equilibrium.

9If we denote by $\psi$ the relative bargaining power of workers, it is easy to show that $\eta = \frac{\psi}{1 - \psi}$.
The optimality condition for the social planner problem gives 10:

\[
\frac{\chi}{AtC_t^{-\sigma}} \leq 1 - (1 + \alpha)Bx_t^\alpha + \beta(1 - \delta)E_t \left\{ \frac{C_t^\sigma}{C_{t+1}^\sigma} \frac{At+1}{At} B(x_t^\alpha + \alpha x_{t+1}^\alpha (1 - x_{t+1})) \right\}
\]

which must hold with strict equality if \( N_t < 1 \). This condition (together with the resource constraints) determine the optimal employment level in equilibrium. The main difference between (18) and (17) is that in the constrained efficient equilibrium the marginal rate of substitution between consumption and labour is equal to the marginal social rate of transformation, as the social planner successfully internalises all labour market externalities.

Comparing the constrained efficient and the decentralized equilibrium with flexible prices, i.e. eq. (18) and eq. (17) respectively, we can see that - as in Blanchard, Galì (2006) - the two equilibria are identical when two conditions are fulfilled:

1. The mark-up \( \mu = 1 \).
2. The workers’ relative share of the surplus in the Nash bargaining, \( \eta \), coincides with the elasticity of the hiring cost function, \( \alpha \) (Hosios condition).

To better clarify the meaning of the second condition, notice that in decentralized economies each firm sets its optimal amount of hirings, without internalising the effects on other firms with the result that the sum of all individual decisions is conducive to an aggregate suboptimal outcome. Indeed, when the workers’ share of the matching surplus \( \eta \) is small in comparison with the hiring costs elasticity \( \alpha \), recruiting additional workers will be highly profitable and employment will set at a higher level; the opposite must be true in case of an excessive workers’ bargaining power \( ((\alpha - \eta) < 0) \) that reduces the incentive to hiring. It is only under the strict equality \( \eta = \alpha \) (as well as \( \mu = 1 \)) that a decentralised equilibrium ends up to be coincident with the optimal social choice.

In the following, to avoid complications arising from the fact that the steady state unemployment differs in the two cases, we assume that employment is subsidized at a constant subsidy \( V \). More precisely, we assume that the subsidy \( V \) is such that the steady state employment level under the decentralized equilibrium corresponds to the steady state employment under the constrained efficient equilibrium11. Notice that, even if the steady state levels correspond, in the short run the dynamics in the two allocations can (and will) differ.

10 Mathematically, the social planner solution derives from the maximization of the following problem:

\[
\max \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \chi N_t \right\}
\]

s.t.

\[
C_t = A_t N_t - G_t H_t = A_t (N_t - Bx_t^\alpha H_t)
\]

\( 0 \leq N_t \leq 1 \)

Notice that we use the fact that, given simmetry in preferences and technology, the social planner chooses an equilibrium in which \( C_t(t) = C_t \). Moreover, since participation in the labour market, by lowering hiring costs, has no individual costs but some social benefits, the social planner will choose an allocation with full participation. See also Blanchard-Galì (2006).

11 It can be shown that the constant subsidy \( V \) is:

\[
V = \mu \{1 - g(\alpha - \eta)(1 - \beta(1 - \delta))(1 - x)\}
\]

where \( g = Bx^\alpha \). With this subsidy, the real marginal cost faced by firm \( i \) is now given by

\[
MC_t = \frac{1}{V} \left\{ \frac{1}{A_t^\alpha} W_t + Bx_t^\alpha - \beta(1 - \delta)E_t \left\{ \frac{A_{t+1} C_t^\sigma}{A_{t+1} C_{t+1}^\sigma} \right\} Bx_{t+1}^\alpha \right\}
\]
It must be emphasised that in our model economy - where variations of intensive margin are replaced by variations of extensive margin - exogenous shocks cause changes in the number of hirings and in recruitment costs. Additionally, the successful matching and the related surpluses offer more opportunities for wage premia and cause a departure of wage rates from competitive values. Indeed, in all the cases where \( \alpha \neq \eta \) one obtains, as seen before, a deviation of the natural equilibrium from the efficient allocation. Furthermore, the bargaining process, for low values of \( \eta \), introduces a substantial degree of wage rigidity (as already mentioned); this effect is not counterbalanced by the reservation wage component, that under the extensive margin hypothesis, exhibits, as we will see better below, a lower elasticity to changes in real activities.

4.1 About output gaps

Blanchard, Galí (2005) argue that the divine coincidence is due to the fact that the gap between the efficient level of output and the natural level of output (i.e. the one that would prevail if prices were flexible) is constant. They also argue that the divine coincidence breaks down in the presence of real wage rigidities.

In this section we show how in our model, even in the absence of real wage rigidities, the gap between the efficient and the natural level of employment (or output) is not constant. The divine coincidence does not hold. The reason is very simple: as the presence of hiring frictions introduces externalities (both firms and workers do not realize the effects that their decisions have on the labour market tightness indicator and thus on aggregate hiring costs) the decentralized equilibrium will not move efficiently. As we shall see in section 5.1 this is the premise for the breakdown of the divine coincidence.

Let variables with bars and the subscript 1 denote log deviations from the steady state of the endogenous variables under the efficient outcome while \( \hat{a}_t \) is an exogenous productivity shocks. Log-linearizing equation (18), we get the evolution of the constrained efficient level of employment, which is implicitly determined as\(^{12}\):

\[
\frac{X}{C} \sigma \bar{\delta} \hat{x}_t \bar{t} = \frac{X}{C} \sigma (\sigma - 1) \hat{a}_t - (1 + \alpha) \alpha g [\hat{x}_t \bar{t} - \beta (1 - x) E_t \hat{x}_{t+1}]
\]

where \( g = B x^\alpha \) and the variables without time subscripts denote steady state values. It can be shown that the constrained efficient employment level will be invariant to productivity shocks if and only if \( \sigma = 1 \). Intuitively, when \( \sigma = 1 \) the first term on the right hand side of (19) vanishes and the productivity shock has no effect on the labour market tightness and hence both \( \hat{x}_t \) and \( E_t \hat{x}_{t+1} \) are equal to zero.

Similarly, it is possible to find the evolution of natural employment level (the subscript 2 denote

\(^{12}\)To derive this solution, we use the fact that the loglinear approximations for the labor market tightness \( \bar{x}_t \) and for consumption are given by:

\[
\delta \hat{x}_t = \bar{n}_t \bar{t} - (1 - \delta) (1 - x) \bar{n}_t - 1
\]

\[
\hat{c}_t = \hat{a}_t + \frac{1}{1 - \delta g} \bar{n}_t + \frac{g (1 - \delta)}{1 - \delta g} \bar{n}_t - 1 - \frac{\varphi g}{1 - \delta g} \delta \hat{x}_t
\]

Following Blanchard-Galí (2006), we introduce two approximations that considerably simplify the characterization of the equilibrium:

1. Hiring costs are small relative to output, so that we can approximate \( \hat{c}_t \) with \( \bar{n}_t \). More precisely, we assume that \( \delta \) and \( g \) are of the same order of magnitude of \( \bar{n}_t \), implying that terms involving \( g \bar{n}_t \) or \( \delta \bar{n}_t \) are of second order. We assume the same to be true for \( \hat{a}_t \) (i.e. \( \hat{a}_t \) and \( \bar{n}_t \) are of the same order).

2. Fluctuations in \( \bar{x}_t \) are large relative to those in \( \bar{n}_t \), an approximation that follows from the log-linearization of the labor tightness index and the assumption of a low separation rate. This implies that terms involving \( g \bar{x}_t \) or \( \delta \bar{x}_t \) cannot be ignored.

Similar approximations are used in the derivation of the flexible prices and sticky prices cases.
the natural level of a variable):

$$\frac{X}{C-\sigma} \frac{\xi}{C-\sigma} \sigma \tilde{n}_{2t} = \frac{X}{C-\sigma} (\sigma - 1) \hat{a}_t - (1 + \eta) \alpha g \tilde{x}_{2t} + \beta g [\alpha + \alpha \eta (1 - x) - \eta x] E_t \tilde{x}_{t+1}$$  \hspace{1cm} (20)

Expression (20) is very similar to (19). Again, in (20) if \( \sigma = 1 \) employment will be invariant to productivity shocks. The reason, in both cases, is that the income effect and the substitution effect on labour supply offset one another. But there is one key difference between the two solutions: while in the constrained efficient solution the social planner correctly internalises the effects of additional hirings on hiring costs - and the elasticity of (shadow) wages to labour market tightness changes is \( \alpha - \) in the decentralised solution workers and firms do not internalise this effect and the elasticity of wages to labour market tightness changes is given by the workers’ relative share of the employment rent, \( \eta \).

We can also express the evolution of natural employment as deviations from the efficient level. Define the employment gap as \( \tilde{n}_t = \tilde{n}_{2t} - \tilde{n}_{11} \). After some algebra we get the evolution of the gap between first and second best employment:

$$\tilde{n}_t = -(1 + \eta) \alpha g \tilde{x} + \beta g [\alpha + \alpha \eta (1 - x) - \eta x] E_t \tilde{x}_{t+1} + (\alpha - \eta) \frac{q}{g} \tilde{\pi}_t$$  \hspace{1cm} (21)

where \( q = \frac{C-\sigma}{\sigma} \), \( \tilde{T}_t = k_0 \tilde{n}_{11} - k_1 E_t \tilde{n}_{11} - k_2 \tilde{n}_{1t-1} \) varies with exogenous shocks and the parameters \( k_i \) depend on structural parameters. Interestingly, as long as \( \alpha \neq \eta \), the gap between the constrained efficient and the natural employment levels is not constant but varies with shocks (through variations in \( \tilde{T}_t \)). That means that even in absence of real wage rigidities, the conditions under which the divine coincidence holds are not met. As already stressed, the reason for this is the presence of a labour market externality: in the decentralised economy firms do not internalise the effects that their hiring decisions have on the aggregate hiring costs. The presence of this dynamic externality means that - even if the steady state of the constrained efficient economy and the decentralised one are the same - the dynamics of the two economies are different. The presence of an externality - even a dynamic one - implies a market failure and is a prima facie reason for policy intervention. As we shall see in the following sections this has profound implications for the trade offs the Central Bank has to face when designing its monetary policy.

5 Introducing Sticky Prices

We now introduce nominal rigidities using the formalism à la Calvo (1983). In each period, firms may reset their prices with a probability \( \zeta \) (independent of the time elapsed since the last revision of prices). The expected time over which the price is fixed is therefore \( \frac{1}{1-\zeta} \). The remaining fraction \( \zeta \) of firms are not allowed to adjust prices.

Log-linearizing around a zero inflation steady state the optimal price setting rule\(^\text{13}\) and the price index equation \( P_t = \left[(1 - \zeta)(P_t^*)^{1-\epsilon} + \zeta (P_{t-1})^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \), we get the New Keynesian Phillips curve:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \lambda \hat{m}_c$$  \hspace{1cm} (22)

where \( \hat{\pi}_t \) denotes consumer prices’ inflation, \( \lambda = (1 - \beta \zeta) (1 - \zeta) / \zeta \) and \( \hat{m}_c \) represent the log deviation of real marginal cost from its steady state value. Note that, while (22) looks like a

\(^{13}\)It can be shown that the optimal price setting rule for a firm resetting prices in period \( t \) is given by:

$$E_t \left\{ \sum_{s=0}^{\infty} \zeta^s Q_{t+t+s} Y_{t+t+s} \left( P_t - \frac{1}{\epsilon - 1} P_{t+t+s} MC_{t+t+s} \right) \right\} = 0$$

where \( P_t^* \) denotes the price newly set at time \( t \), \( Y_{t+t+s} \) is the level of output in period \( t+s \) for a firm resetting its price in period \( t \) and \( \epsilon \) is the gross desired markup. \( Q_{t+t+s} = \beta^s \left( \frac{C_t}{1+\epsilon} \right) \) is the stochastic discount factor for nominal payoffs.
standard New Keynesian Phillips curve, the dynamics of the real marginal costs are now substantially different from the ones of a standard NK model, as they are deeply affected by the labour market institutions. In fact, it is possible to show that the dynamics of marginal costs in terms of deviations from the first best take the following form:

\[
\frac{\nu}{\mu} \tilde{m}c_t = \frac{\chi}{C_\sigma} \tilde{n}_t + (1 + \eta) \alpha g \tilde{x}_t - \beta g [\alpha + \alpha \eta (1 - x) - \eta x] E_t \tilde{x}_{t+1} \\
- (\alpha - \eta) \frac{g}{\delta} \bar{T}_t
\]

(23)

where variables with tilda are deviations of the sticky price outcome from the constrained efficient allocation.

Equation (23) highlights the determinants of marginal costs. Marginal costs increase with the number of the employed (\(\tilde{n}_t\)) as the firm has to pay higher wages to persuade households to provide more labour. This is the only channel at work in the standard NK model. In the hiring cost model, the changing of labour market conditions at time \(t\) (i.e. an increase of \(\tilde{x}_t\)) increases marginal costs through two channels. A more tight labour market, in fact, increases both hiring costs and the bargained wage, as the rents associated to an existing employment relationship are higher. An expected increase of \(E_t \tilde{x}_{t+1}\), instead, has the opposite effect, as it becomes convenient for the firm to hire at time \(t\) in order to be ready for a more difficult labour market in time \(t+1\). Finally, in the presence of the labour market externality (i.e. when \(\alpha \neq \eta\)), productivity shocks have an independent effect on marginal costs: the divine coincidence does not hold.

5.1 The Divine Coincidence breaks down

In this model, the presence of labour market frictions introduces a non-trivial trade-off between inflation and output gap stabilization: the divine coincidence does not hold. To see this more clearly, consider at first a central bank that adopts a “pure inflation targeting” strategy, i.e. a strategy aimed at stabilising inflation at all horizons (\(\bar{\pi}_t = 0\) for all \(t\)). From (22) and (23) it follows that the employment gap evolves according to the following:

\[
\tilde{n}_t = -(1 + \eta) \alpha g \tilde{x}_t + \beta g [\alpha + \alpha \eta (1 - x) - \eta x] E_t \tilde{x}_{t+1} + (\alpha - \eta) \frac{g}{\delta} \bar{T}_t
\]

(24)

where \(\bar{T}_t\) (as long as \(\sigma \neq 1\)) is positively related to productivity shocks.

The important point to note is that here - in contrast to the standard NK model - a pure inflation targeting strategy is unable to stabilise the employment gap in the face of productivity shocks: employment deviations from the benchmark will be sizeable and display a high degree of inertia. Notice also that under the pure inflation targeting strategy, firms have no incentive to change their prices\(^{14}\); accordingly, the dynamics of the employment gap replicate exactly the dynamics under flexible prices, as can be easily seen by comparing (24) with (21).

Secondly, consider a “pure employment targeting” policy, a strategy aimed at stabilising the (un)employment gap in each period, i.e. \(\tilde{n}_t = 0\) for all \(t\), which implies that also the labour market tightness is constant (\(\tilde{x}_t\)) for all \(t\). Iterating forward the Phillips Curve (22), one gets:

\[
\bar{\pi}_t = - (\alpha - \eta) \frac{g}{\delta} \sum_{s=0}^{\infty} \beta^s E_t \hat{\bar{\pi}}_{t+s}
\]

A “pure employment targeting” strategy is thus unable to stabilise inflation. The presence of hiring costs, by affecting the distance between the constrained efficient and the natural level of output, creates a non trivial trade-off between output and inflation stabilisation. This calls into question the role of the monetary authority.

\(^{14}\)See e.g. Galí (2002) for a discussion of this point.
We remark that either one or the other of the following conditions must be satisfied for the divine coincidence to hold:

1. The intertemporal substitution parameter, $\frac{1}{\psi}$, must be equal to 1, i.e. $\sigma = 1$. If $\sigma = 1$, in fact, the income and substitution effects on labour supply offset each other and employment is invariant to productivity shocks. Hence, as the dynamics are closed, the dynamic externality cannot have any impact. Mathematically, $T_i = 0$.

2. The elasticity of hiring costs to labour market tightness, $\alpha$, is equal to the relative bargaining power of workers, $\eta$ (Hosios condition). When $\alpha = \eta$ the labour market externality (and consequently the dynamic trade-off between inflation and unemployment stabilization) disappears.

For the divine coincidence not to hold $\sigma \neq 1$ and $\alpha \neq \eta$ must simultaneously apply. Interestingly, the direction of the trade-off is not un controversial, since it depends on the hiring costs frictions and on the workers’ bargaining power.

Consider for instance equation (24). Three cases can be considered. When $\alpha > \eta$, a positive productivity shock generates a positive employment gap, i.e. employment under the decentralized solution reacts more than under the efficient solution. Intuitively, the hiring cost elasticity $\alpha$ determines the efficient response of wages to hiring costs, while the workers’ share in the Nash bargaining, $\eta$, determines how wages evolve in the decentralized equilibrium. When $\alpha > \eta$, the response of wages to shocks in the decentralized equilibrium is inefficiently low, and employment reactions is too strong, giving rise to a positive employment gap. When $\alpha < \eta$ the opposite is true and a productivity shock creates a negative employment gap. In the limiting case $\alpha = \eta$, real wages react exactly as they would in the constrained efficient solution, the labour market externality disappears and the divine coincidence holds again\(^{15}\).

The presence of the labour market externality is thus likely to have interesting policy implications. To the extent that the central bank values both inflation and unemployment stabilization, a pure inflation targeting strategy is not optimal anymore. The direction of the interest rate response, however, depends crucially on the labour market characteristics. Consider again a positive productivity shock. When $\alpha > \eta$, real wages do not react enough to productivity shocks and the central bank can artificially increase wage reaction by creating negative inflation, thus reducing the employment gap. Similarly, when $\alpha < \eta$ the employment gap can be reduced by allowing positive inflation, which tends to stabilise real wages. Optimal monetary policy is thus likely to be pro-cyclical when the workers’ bargaining power is low and counter-cyclical when it is high\(^{16}\).

6 The Monetary Authority and the reduced model

In order to close the model, we need first to introduce a suitable characterisation of monetary policy. As in the present paper we are not interested in pursuing an analysis of optimal monetary policy, we shall simply assume the Central Bank sets the short term nominal interest rate by reacting to the average inflation and employment gap levels in the economy. Specifically, we assume the monetary authority follows the Taylor-type rule:

$$(1 + i_t) = \beta^{-1 - \rho_m} (1 + i_{t-1})^{\rho_m, \pi_x (1 - \rho_m) (n_t)^{\phi_x (1 - \rho_m)} }$$

Log-linearising it around the steady state, one can get:

$$\hat{i}_t = \rho_m \hat{i}_{t-1} + \phi_x (1 - \rho_m) \hat{n}_t + \phi_y (1 - \rho_m) \hat{\pi}_t + \epsilon_t^m$$ (25)

\(^{15}\)To get additional intuition about the results, a simple mapping between this model and the standard labour market search model can also be used. It is easy to show that the hiring cost elasticity $\alpha$ corresponds to $\frac{1}{1 - \psi}$ in the standard search model, where $\xi$ is the elasticity of the matching function with respect to unemployment. As we mentioned before, the workers’ share in the Nash bargaining can be written as $\eta = \frac{1}{1 - \psi}$, where $\psi$ is the workers’ relative bargaining power. Since empirical estimates of $\xi$ are close to 0.5, we can consider $\alpha$ to be around 1. It follows that $\eta > \alpha$ if workers’ bargaining power is higher than firms’ bargaining power (i.e. $\psi > 0.5$) and vice versa.

\(^{16}\)See Faia (2007) for similar results in a much richer model.
Consistently with empirical evidence, we assume that monetary policy displays a certain degree of interest rate smoothing\(^{17}\). The parameters \(\phi_\pi\) and \(\phi_y\) are the response coefficients to inflation and the employment gap respectively. The term \(\varepsilon_t^m\) captures an i.i.d monetary policy shock.

The model presented so far, although featuring several market imperfections and institutional parameters, can be reduced to a relatively simple three equations macro-model as can be done with the standard NK model. The equilibrium in our economy with hiring costs, Nash bargaining and equilibrium unemployment is fully characterised by the Euler equation (that gives rise to the IS equation), the NKPC and the description of monetary policy. Using the approximation \(\tilde{u}_t = -(1-u) \tilde{n}_t\), we can write the system as deviations from steady state values as\(^{18}\):

1. **IS**

\[
\tilde{u}_t = \frac{(1-u)}{\sigma}(\tilde{\pi}_t - E_t \tilde{\pi}_{t+1}) + E_t \tilde{u}_{t+1}
\]  

(26)

2. **Monetary Policy**

\[
\tilde{\pi}_t = \phi_\pi \tilde{\pi}_{t-1} + \phi_\pi (1 - \rho_m) \tilde{\pi}_t - \frac{\phi_\pi (1 - \rho_m)}{(1 - u)} \tilde{u}_t + \varepsilon_t^m
\]

(27)

3. **NKPC**

\[
\tilde{\pi}_t = \beta E_t \{\tilde{\pi}_{t+1}\} - \kappa_0 \tilde{u}_t + \kappa_1 \tilde{u}_{t-1} + \kappa_2 E_t \tilde{u}_{t+1} - \kappa_3 \tilde{a}_t
\]

(28)

The main difference with respect to a standard NK model - apart from the fact that here we allow for involuntary unemployment - lies in the NKPC, where the coefficients \(\kappa_i\) are now functions of the structural parameters characterizing the two economies: workers’ bargaining power, hiring costs, separation rates, markups, degree of nominal stickiness, and so on. Intuitively, the introduction of hiring costs frictions substantially change the dynamics of the marginal costs, which in turn influence the firms’ optimal price setting and thus the slope of the Phillips curve.

Something more is to be said about the Phillips curve in (28). The current inflation unemployment trade off is conditioned by \(\kappa_0\) and this parameter is defined as follows:

\[
kappa_0 = \kappa_0' + \kappa_0''
\]

\[
kappa_0' = \frac{\mu}{\sqrt{V}} \frac{1}{(1-u)^{\frac{1}{2}}} g \sigma
\]

\[
kappa_0'' = \frac{\mu}{\sqrt{\delta}} \frac{1}{(1-u)} \left(1 + \eta\right) g \alpha + \beta g \left(\alpha + \alpha \eta \left(1 - x\right) - \eta x\right)
\]

where \(g = \frac{\chi}{C^\frac{1}{\sigma}}\). Furthermore, for the lagged and expected terms one has the parameters:

\[
kappa_1 = \frac{\mu}{\sqrt{\delta}} \frac{1}{(1-u)} \left(1 + \eta\right) g \alpha \left(1 - \delta\right) \left(1-x\right)
\]

\[
kappa_2 = \frac{\mu}{\sqrt{\delta}} \frac{1}{(1-u)} \beta g \left(\alpha + \alpha \eta \left(1 - x\right) - \eta x\right)
\]

\[
kappa_3 \equiv \frac{\sigma}{\beta}
\]

\(^{17}\)See, e.g., Clarida, Gali and Gertler (1999).

\(^{18}\)Notice that, to perform simulations, we have used the model derived in terms of deviations from steady state instead that in terms of deviations from the first best. We decided so mainly for two reasons. First, the focus of the analysis is positive, and the model in terms of deviations from steady state is more directly comparable with actual data (especially after that both have been detrended in the same way). As many have highlighted, in fact (see e.g., Gali 2002), the theoretical output gap has a different nature than the HP-detrended output gap (and is much more difficult to estimate). Second, the results of our model are more easily comparable with others found in the literature.
As one can see from \( \kappa_0 \) and \( \kappa_0^* \), the slope coefficients which capture the current inflation unemployment tradeoff are determined by the interaction of two groups of structural parameteres: \( \lambda \), which is influenced by the nominal rigidity indicator \( \zeta \), and various structural imperfections of product (\( \mu \)) and labour markets (\( \alpha, \eta, \delta \)).

The tradeoff is thus governed by the sensitivity of price adjustments to nominal shocks, but also by all the factors that enhance the complementarities in price decisions and thus amplify the macroeconomic impact of price stickiness at the single firm level. First of all, a pricing complementarity (obtained by higher values of \( \mu \)) reduces the incentive of a single firm to change its relative price and thus it is a source of stickiness which enters as a multiplicative factor in \( \kappa_0 \) and \( \kappa_0^* \). Secondly, all the labour institutions that enhance real wage rigidities reduce the individual incentive to price adjustment; in other words, in front of a demand shock requires a smaller change in relative price, the smaller is the change of the real wage. This means that all the variables which cause a flat wage curve and a flat supply curve strengthen the role of \( \zeta \), making it more likely a non vertical Phillips curve.

A more detailed analysis of such structural labor market imperfections shows that the magnitude of hiring frictions and their combined influence (the multiplicative terms in \( \eta, g, \alpha \) in \( \kappa_0^* \)) play a role in explaining the inflation unemployment trade off. Let us consider, for instance, the propagation mechanism in case of a monetary shock. Under positive nominal shocks, as output raises and firms increase hirings, the labour market tightens. However, unemployment reductions are smaller the higher are the hiring costs and matching rents going to employees. Thus higher values of \( g, \alpha, \eta \) (through \( \kappa_0^* \)) are accompanied by smaller decreases in unemployment and a more steep Phillips curve is obtained.

Additional considerations relate to the role of firing restrictions, negatively associated with the separation rate: when the probability of leaving the firm \( \delta \) is high, an exogenous shock makes employment more sensitive to current labour market conditions; a low value of \( \delta \) implies that more jobs survive from one period to the next one, and it represents a serious obstacle for a high degree of adjustment of current labour demand that ends to generate a steeper Phillips Curve.

Furthermore, the inflation unemployment trade off is affected by the job finding rate \( x \), as detectable by the expression for \( \kappa_0 \). A higher value of \( x \) means that recruiting efforts of employees have more chances to actually turn into effective jobs; however, a high value of \( x \) has also an alternative meaning since it represents a situation of labor market tightness that reinforces the wage premia. A demand expansion that feeds inflation is thus capable to deliver unemployment contractions the higher is \( x \), thus the Phillips Curve of our model economy is flatter, the higher is \( x \), as we will further examine in the next section.

Finally, hiring frictions influence the intertemporal inflation-unemployment relation, a dynamic linkage totally absent in the standard New Keynesian Phillips Curve. As shown by (29), past and expected unemployment rates influence current inflation rates and, under positive recruiting costs, even if price-setters are not backward-looking, current inflation rates are in the end influenced by past hiring decisions. A positive link between lagged unemployment rates and current marginal costs (and inflation rates) may be explained as follows. For a given amount of current labor demand \( N_t \), the higher is the labour workforce inherited by the previous recruiting efforts (and thus the lower is the unemployment rate), the lower are the today hirings needs, since \( H_t = N_t - (1-\delta)N_t^{i-1} \); moreover, for a given level of desired hirings, the lower is the unemployment rate, the higher are the congestion effects, since \( -\frac{\partial u_t}{\partial N_t} = -(1-\delta) \) and \( G_t = A_t B \left( \frac{H_t}{\bar{t}} \right)^{\alpha} \). Therefore, the model with
hiring costs is suitable to replicate the persistent response of output and employment.$^{19}$

The positive relation between *expected* unemployment and *current* inflation is reasonable by simply recalling, as seen in section 3.2.2, that current marginal costs decrease with the expected labor tightness and thus one can easily justify $\kappa_2 > 0$. Both the dynamic effects, the lagged and the expected ones, are more significant when workers and employees are tied by long term relationships, as can be ascertained by inspecting the values of $\kappa_1$ and $\kappa_3$ which become larger for lower separation rates $\delta$.

In the following, we focus on the positive implications of different labour market institutions for the dynamic behaviour of the stylized economy. We first compare the model with a standard New Keynesian model, where structural imperfections in the labour market are absent. We then perform a sensitivity analysis in order to explore how the economy responds to shocks as some fundamental parameters change. The model presented allows one to pursue the analysis of the differences in dynamic performance between two economies, characterised by different degrees of market imperfection and labour protection.

### 7 Calibration

In this section we describe the parameter values used in our baseline calibration. These parameters are chosen to be largely consistent with those standard in the New Keynesian literature. The following table summarises the baseline values for the key parameters of our model with hiring costs:

<table>
<thead>
<tr>
<th>Preferences and Technology</th>
<th>$\beta$</th>
<th>$\sigma$</th>
<th>$\epsilon$</th>
<th>$\mu$</th>
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<tr>
<td></td>
<td>0.99</td>
<td>2</td>
<td>11</td>
<td>1.1</td>
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<tr>
<td>Labour market</td>
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<td></td>
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<tr>
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<td>0.7</td>
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<td>$x$</td>
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<tr>
<td>$\delta$</td>
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<tr>
<td>$\eta$</td>
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<td>$\phi_y$</td>
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<td>1.1</td>
<td>0</td>
<td></td>
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<tr>
<td>Shocks’ Persistence and volatility</td>
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<td>$\rho_a$</td>
<td>$\sigma_a$</td>
<td>$\sigma_m$</td>
</tr>
<tr>
<td></td>
<td>0.0</td>
<td>0.9</td>
<td>0.01</td>
<td>0.002</td>
</tr>
</tbody>
</table>

*Preferences and technology:* $\beta$ is set equal to 0.99, which implies a riskless annual return of about 4 percent (The time period is taken to correspond to a quarter). We assume $\sigma = 2$, which implies a higher degree of risk aversion than that implied by a log utility function. The elasticity of substitution between differentiated goods $\epsilon$ is set equal to 11, corresponding to a markup $\mu = 1.1^{20}$. The steady state level of productivity $A$ is set equal to 1 only for simplicity.

*The labour market:* In the baseline calibration, we set unemployment to be $u = 0.05$, which is roughly consistent with the average unemployment in the US. The steady state job-finding rate $x$ is set to 0.7, which corresponds approximately to a monthly rate of 0.3. Given $u$ and $x$, it is possible to determine the separation rate using the relation $\delta = ux / ((1 - u)(1 - x))$. We obtain a value for $\delta$ roughly equal to 0.12. The relative bargaining power $\eta$ is set to 0.5, which implies that

---

$^{19}$By simply rewriting the New Keynesian Phillips Curve obtained in our model economy, one gets the persistent and expected effects that influence the current unemployment rate:

$$
\hat{u}_t = \frac{1}{\kappa_0} \hat{u}_t - \frac{1}{\kappa_0} \beta E_t \{\hat{u}_{t+1}\} + \frac{\kappa_1}{\kappa_0} \hat{u}_{t-1} + \frac{\kappa_2}{\kappa_0} E_t \hat{u}_{t+1} - \frac{\kappa_3}{\kappa_0} \kappa_3 \hat{u}_t
$$

$^{20}$Notice that a mark-up of 1.1 is definitely lower than the average (1970-1992) mark-up in manufacturing estimated for several OECD countries by Oliveira Martins, Scarpetta, Pilat (1996).

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firms have higher bargaining power than workers. The scaling parameter $B$ is chosen such that hiring costs represent a 1 percent fraction of steady state output\(^21\). The parameters $\chi$ can then be determined using steady state identities.

The degree of price rigidity $\theta$ is set equal to 0.75, as in Galí (2002), implying an average duration of a price contract of one year (a level higher than that suggested in Galí and Gertler, 1999 for the U.S. economy).

Following Walsh (2005), we adopt a baseline interest rate rule for monetary policy where the central bank is assumed to respond to inflation but not to the economic activity ($\phi_x = 0$). Furthermore, we assume that the degree of inertia in the policy rule $\rho_m$ equals 0.9, a value consistent with the empirical evidence on policy rules.

Persistence and volatility of shocks: productivity shocks have a persistence parameter $\rho_a = 0.9$. Following Walsh (2005), we set the standard deviation of the policy shock $\sigma_m = 0.002$ and the standard deviation of productivity shocks to $\sigma_a = 0.01$.

8 Comparison with the standard NK model

In this section, we compare the dynamics of our model with hiring costs with that of a standard New Keynesian model. Notice that, for an easier comparison, we use exactly the same parameter values for the two models. The only parameter that enters into the standard NK model, but is absent in our model, is the inverse of the elasticity of labour supply, $v$, which - consistently with micro evidence - we set equal to 3. The introduction of hiring frictions substantially modifies the working of the labour market and the transmission mechanism of shocks.

In the standard NK model the labour market is perfectly competitive and labour supply and demand meet to clear the market. In the hiring costs model the presence of hiring frictions introduces two opposite effects. On the one side, labour demand becomes steeper. To see this, consider the evolution of marginal costs equation (15), which we rewrite here for convenience:

$$MC_t = \frac{W_t}{A_t P_t} + \left\{ B x_t^\alpha - \beta (1 - \delta) E_t \left( \frac{A_{t+1}}{A_t} \right)^{\sigma} B x_{t+1}^{\sigma} \right\}$$

In the hiring costs model, marginal costs depend not only on real wages and productivity (as in the standard NK model), but also on marginal hiring costs, which are increasing in $x_t = H_t$. Increasing the number of employed in the economy thus raises hiring costs and makes the adjustment on the labour quantity side more costly. When $U_t$ is very small, the labour demand schedule becomes almost vertical as hiring costs become prohibitively high.

On the other side, the wage rule with hiring costs is flatter than the labour supply function in a standard New Keynesian model. By passing from the intensive to the extensive margin of labour variations the reservation wage becomes more rigid and permits to lower the elasticity of real wages with respect to changes in output. This second channel tends to limit changes in the price side and to increase the fluctuation of quantities.

These two effects operate in opposite directions and tend to offset each other. It can be shown that for reasonable parameter values (that is, as long as the elasticity of disutility of labour to hours worked $v > 1$), the second effect tends to dominate, and the model with hiring frictions is found to have more rigid real wages\(^22\).

\(^{21}\)To pin down $B$, we use the fact that in steady state hiring costs represent a fraction $\delta g = \delta B (x)^\alpha$ of GDP.

\(^{22}\)It must be reminded that in the hiring costs model the reservation wage is given by $\frac{1}{x_t} = \frac{L_t}{h_t}$, thus the slope of the labour supply is conditioned by $\frac{1}{1 + v}$, where $h$ is the fixed amount of working hours. On the contrary, in the standard NEK model, with utility function (1), the slope of the labour supply is conditioned by $\xi h v$, where hours $h$ are elastically supplied.
Figure 1 shows the impact of a monetary shock, which in our simulation takes the form of a 1% increase in the nominal interest rate.

Several interesting facts emerge. First, inflation in the model with hiring costs appears to be less volatile and slightly more persistent than in a standard NK economy. Second, the response of employment shows higher persistence in the hiring model. Therefore, the model with hiring costs is able to better replicate a central dynamic feature of real world economies, namely “the sluggish response of inflation together with the large and persistent response of output” (Trigari 2005, p. 2). Third, in the hiring-model the sensitivity of real marginal costs and of real wages to output changes is much lower than in the standard NK model. Interestingly, the low volatility of real wages is obtained endogenously, without the need to impose an unexplained real wage rigidity.

The intuitive reasons behind the results here obtained are as follows: a positive nominal shock causes an increase in the aggregate demand for goods and labour. Accordingly, in period $t$ recruiting activities and unit hiring costs also increase. However, for each additional hiring undertaken in this period, there will be $(1 - \delta)$ more employed workers in the next period. In this context, additional current hirings generate, in period $t + 1$, two externalities. On the one hand, through the thick market effect, the increase in the number of employed workers reduces the costs of new hires; on the other hand, a lower level of unemployment has a negative impact since it represents an obstacle to the matching process and thus increases hiring costs (thin market externality). These two forces - which influence the labour market tightness index - may counterbalance one another and as a net effect may produce not only a less pronounced responsiveness of marginal costs to employment fluctuations, but also a smoother dynamics.

It must be noticed, as shown in Figure 1, that if the wage contraction is of limited magnitude, the marginal cost response is wider, since it includes the contraction due to savings in hiring costs; thus the gap between wage and cost changes, as analysed in section 3.2.2, may contribute to explain the low correlation between inflation and real wages observed in reality (Krause and Lubik, 2003). In any case, from the comparison of the two models shown in Figure 1, it is relevant to stress a higher degree of rigidity in marginal costs in the hiring model with respect to the parallel response recorded in the standard NEK model, partially explained by the reversal impacts of expected hiring costs, analyzed in section 3.2. These divergent patterns are prolonged and last in subsequent periods.

Our simple model with hiring costs is thus able to overcome many of the dynamic weaknesses of the standard NK model. Furthermore, it can be shown that these dynamics, obtained with a simple and tractable model, are qualitatively similar to the ones obtained in those far more complex NK models which incorporate labour search (see e.g. Trigari, 2005 or Walsh, 2005).

Figure 2 shows the impact of a positive productivity shock in the hiring costs model and in the standard NK model. The presence of hiring frictions substantially affects the dynamics of the real wages. In the standard NK model real wage deviations decrease on impact but turn positive in the medium run; in the hiring costs model, instead, real wages react much less on impact and remain negative in the medium run. Accordingly, marginal costs and inflation are less volatile in the model with hiring costs. Finally, the presence of hiring frictions explains why employment reacts by less in the hiring costs model than in the standard NK.

9 Inflation and Employment Volatility and Labor Market Institutions

The objective of this sections is to analyse how inflation and unemployment volatility are influenced by different labour market structures. Calibrating the degree of labour market rigidities is a challenging task, as the overall degree of “rigidity” in the labour market does not depend only on one parameter but on all the configuration of the labour market, as captured by the interplay of different parameters. It is common in the literature to vary one parameter at a time; this strategy however has the drawback that it ignores the fact that in real world economies different
Figure 1: Monetary Policy Shock

Figure 2: Productivity Shock
parameters are usually linked in an intuitive and predictable way. Here we analyse how inflation and unemployment volatility are influenced by different labour market institutions by taking explicitly into account this fact.

Specifically, following Blanchard and Galí (2006), we characterize the structure of the labour market by calibrating the steady state unemployment and job-finding rates \((u\) and \(x\)); the separation rate is then determined through the steady state relationship \(\delta = ux/(1 - u)(1 - x)\). Figure 3 shows how inflation and unemployment volatility depend on the job-finding and unemployment rates. Simulations are performed by varying the job-finding rate from 0.25 to 0.75 and the unemployment rate from 0.11 to 0.05. Each point in the \((x, u)\) plane corresponds to a different steady state (i.e. to a different stylised economy); the vertical axes displays the corresponding level of inflation and unemployment volatilities. The lower corner corresponds to a very “rigid” country, i.e. a country that has a low job-finding rate (0.25) and a high unemployment rate (0.11). The opposite holds for the upper corner. Both \(x\) and \(u\) go therefore from the more rigid to the more flexible case.

\[\delta = \frac{ux}{(1 - u)(1 - x)}\]

Figure 3: Labour Market Institutions and Volatilities

Figure 3 allows one to analyse how the inflation and unemployment volatility of the stylized economy change when the job-finding rate and/or the unemployment rate vary, taking as given the monetary policy rule and the nature of exogenous shocks.

Consider at first the \textit{coeteris paribus} effect of an increase in the job-finding rate \(x\). For a given

\[\text{We calibrate directly the job-finding rate and the unemployment rate because these are more easily estimated than the reservation wage or the separation rate.}\]

\[\text{Inflation and unemployment volatilities are obtained by simulating the model using both monetary shocks and productivity shocks. The persistence and volatilities of shocks are calibrated as explained in section 7}\]
level of the unemployment rate (for example $u = 0.05$), inflation volatility varies between more than 1.1 when the job-finding rate is 0.3 to less than 0.7 when $x = 0.75$. Unemployment volatility instead, for the same parameters' values, goes from 0.55 to 1.1. In other words, a higher job-finding rate reduces the volatility of inflation and increases unemployment volatility. Intuitively, if the job-finding rate is higher, workers face a better outside option (i.e. they can find more easily another job) and they are therefore less willing to accept a big reduction in wages in order to keep the job. The volatility of real wages, marginal costs and inflation is thus inversely related to the job-finding rate; the opposite holds for the volatility of unemployment.

Interestingly, the coeteris paribus effect of a decrease in the steady state unemployment rate $u$ is opposite. For instance, for a job-finding rate fixed at $x = 0.3$, inflation volatility increases from below 0.8 to more than 1.1 when $u$ goes from 0.11 to 0.05; unemployment volatility instead decreases from almost 1 to less than 0.6. The intuition behind this result is as follow. In the model, coeteris paribus, a lower unemployment can only be explained - through steady state relationships - by a lower steady state reservation wage $\frac{\chi}{A_1C_1}$ and a lower probability of job termination $\delta$. A lower steady state reservation wage implies a lower degree of real wage rigidity, since the portion of the wages that is relatively insensitive to market variations is relatively smaller; when the probability of exogenous separation gets lower, instead, more matches survive from one period to the other and employment becomes less sensitive to labour market conditions. These two channels thus go in the same direction, and explain why, in our model, the volatility of unemployment is positively correlated with the steady state unemployment rate.

![Figure 4: Trade-off between inflation and unemployment volatility](image)

Figure 4 shows the implications of different labour market structures for the tradeoff between unemployment volatility and inflation volatility (simply calculated as the ratio among the two). Different labour market institutions determine how costly it is for firms to absorb shocks by changing prices or by changing the quantities produced. A higher job-finding rate, which makes the adjustment on the quantity side easier, tends to increase unemployment volatility and to decrease inflation volatility: the tradeoff increases. A lower unemployment rate, which has the opposite
effect, reduces the unemployment/inflation tradeoff.

In a more rigid economy, which is typically characterized by a lower job-finding rate but a higher unemployment rate, the two effects tend thus to operate in opposite directions. Which one is likely to dominate?

To answer this question, we start by noticing that it is difficult to imagine economies characterized by a low job-finding rate and a low unemployment rate, and vice versa. Indeed, job-finding rates and unemployment rate seem to be linked in the following way: countries with low job-finding rates are usually those with higher unemployment rate, and vice versa. It is therefore reasonable to assume that real world economies are placed around the North-Sud diagonal in Figure 4.

We therefore focus on this diagonal and construct a “labour market flexibility” index that defines a labour market as flexible when it is characterized by high job-finding and low unemployment rates. Specifically, we take a one dimensional look in which the job finding rate and the unemployment rate are linearly linked. Figure 5 shows the relationship between the job-finding rate $x$, the unemployment rate $u$, and the implied separation rate $\delta$ used in the calibration. Notice that to any particular value of labour market rigidity corresponds a different steady state and that in a rigid economy, as in real data, a low job-finding rate is associated with a low separation rate and a high unemployment rate.

![Figure 5: Labour Market Flexibility Index](image)

The results of the simulations performed by varying the degree of labour market flexibility are in Figure 6. More rigid labour markets tend to increase the volatility of inflation and to decrease the volatility of real variables. The trade-off between inflation and output volatility is therefore decreasing in the degree of labour market rigidities. These two results can be reconciled by looking at the impulse response functions (see the appendix). When labour markets are more rigid, monetary shocks are mainly absorbed through a large (but short-lived) increase in inflation, while both monetary and productivity shocks entail smaller unemployment fluctuations. Intuitively, when hiring new workers becomes more costly, firms find relatively more convenient to absorb a shock through changes in prices than through changes in the produced quantities. As a consequence,

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25 The simulation are performed assuming that the job-finding rate varies between 0.25 and 0.75 and the unemployment rate varies between 0.11 and 0.05.
inflation reacts more to shocks while the response of (detrended) output and unemployment gets smaller.

Figure 6: Rigid vs flexible economies

More sophisticated explanations can be given. As noted before, what we show is a purely general equilibrium result. *Coeperis paribus*, higher job-finding and separation rates both increase unemployment volatility. A higher steady state job-finding rate increases the negotiated wage and the degree of real wage rigidity: inflation becomes less sensitive to unemployment changes (the Phillips curve gets flatter). Similarly, as the probability of exogenous separation gets higher, fewer matches survive from one period to the other and employment becomes more sensitive to labour market conditions. Again, this implies that inflation is less sensitive to unemployment changes. A lower steady state unemployment rate has the opposite effect, as a lower unemployment rate is explained by a lower reservation wage and a lower separation rate, which both tend to decrease unemployment volatility.

As we have just shown, under realistic values of these variables, the first two effects dominate and more “flexible” labour markets tend to be characterized by bigger real fluctuations (as the Phillips curve gets flatter). These findings seem to mimic well the actual experience of the US and Euro area economies, the US being more flexible and displaying more volatile output levels and the Euro area being more rigid and displaying less volatile output levels but more volatile inflation (Giannone, Reichlin, 2006).

10 Conclusion

There is by now a number of papers aimed at overcoming the main shortcomings of the standard New Keynesian model, i.e. the absence of involuntary unemployment, the absence of a trade off between output and inflation stabilisation and the low and persistent output response to shocks. Some of these papers assume both labour market imperfections and an unexplained real wage
rigidity. In the present paper we show that a relatively easy to model labour market imperfection, due to hiring costs (à la Howitt), is sufficient to deliver the results obtained by some more complex recent contributions, without introducing an exogenous real wage rigidity.

We have seen that the introduction of a more realistic labour market structure into an otherwise standard New Keynesian framework is capable of delivering involuntary unemployment as a steady state equilibrium and the breakdown of the so-called divine coincidence. The existence of a dynamic externality - due to the effects of hiring decisions on hiring costs faced by firms - makes the dynamic behaviour of a decentralised economy differ from that of the constrained efficient economy and leads the divine coincidence to vanish. After allowing for staggered pricing à la Calvo, we showed that - due to the absence of divine coincidence - neither is pure inflation targeting able to stabilise employment fluctuations nor is pure employment targeting able to stabilise inflation, as it would be the case in a standard NK model. As a consequence it seems at least unwarranted the so-called “World Consensus on Monetary Policy” claimed by Marvin Goodfriend (2007), according to whom “inflation targeting yields the best cyclical behaviour of employment and output that monetary policy can deliver. Thus, and here is a revolutionary point delivered by the modern theoretical consensus, even those who care mainly about stabilization of the real economy can support a low-inflation objective for monetary policy” (Goodfriend, 2007, p. 26).

In order to test the ability of our model to fit some real-world feature of economic dynamics, we identified a measure of economy-wide labour market rigidity, based on the unemployment rate and on the job finding rate (but not on real wage stickiness). This allowed us to simulate the inflation and output volatility of two economies differing in their degree of labour market rigidity. We found that a “rigid” economy (according to our measure) displays lower unemployment volatility and higher inflation volatility than a “flexible” economy. A result that is broadly consistent with the findings of Giannone, Reichlin (2006) referred to the “flexible” US and the comparatively “rigid” Euro area.

11 References


A  Impulse Responses

Figure 7: