Monetary and Fiscal Policy Interactions with Debt Dynamics∗

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Preliminary and Incomplete

May 28, 2013

Abstract

We analyze the interaction between committed monetary and discretionary fiscal policy in a model with public debt, endogenous government expenditure, distortionary taxation and nominal rigidities. Fiscal decisions lack commitment but are Markov-perfect. Lack of commitment by the fiscal authority generates a unique stable steady-state level of debt that depends on the extent of its time-consistency problem. Larger price adjustment costs as well as a more inefficient level of output reduce the steady-state level of debt. The time-consistent, optimal response to shocks features strategic use of the real interest rate, which is reduced in response to budget deficits and vice versa for surpluses. Lack of commitment in fiscal policy leads to volatile tax rates and inflation. When the monetary authority commits to a Taylor-type rule, the unique stable steady-state level of debt increases with the degree of price stickiness.

Keyword: Monetary and fiscal policy interactions, taxation, public debt, business cycle.

JEL Codes: E24, E32, E52

∗Gnocchi gratefully acknowledges financial support from the Spanish Ministry of Education and Science through grant ECO2009-09847, the support of the Barcelona GSE Research Network and of the Government of Catalonia; Luisa Lambertini gratefully acknowledges financial support from the Swiss National Foundation, Sinergia Grant CRSI11-133058.

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1 Introduction

Characterizing the optimal monetary and fiscal policy mix is a classical problem that the literature has repeatedly faced, finding different solutions depending on the distortions the policy makers have to address, the instruments they have available and the way they interact. In this paper, we reassess this issue in an environment where the Pareto efficient allocation may not be implementable, because markets are imperfectly competitive, prices are sticky, lump-sum taxes are not available and policy makers are independent authorities that do not necessarily share the same ability to commit. As far as policy instruments are concerned, we assume that the central bank is responsible for setting the nominal interest rate, while the fiscal policy maker has access to three instruments: non-contingent nominal debt, distortive taxation on labor and government expenditure.

Parts of our complex policy problem have been studied already in isolation. However, this paper is novel in uncovering some important interactions among them. In particular, in an environment like ours, we show that the optimal response to business cycle shocks and steady state distortions are indissolubly linked through public debt in a way that depends on monetary and fiscal policy coordination. Some authors have admittedly emphasized that optimal monetary policy is affected by whether the economy fluctuates around an efficient equilibrium or not. However, our point, though complementary, is substantially different since our channel goes through debt dynamics and policy coordination. Intuitively, even though monetary policy remains neutral in the long-run, the fiscal policy maker has access to instruments that can permanently change the level around which the economy fluctuates. In addition, the incentives for the fiscal authority of relying on debt and distortive taxation are affected by the central bank’s response to temporary shocks. It follows that the widespread practice of abstracting from debt dynamics and steady state distortions may be very misleading. On the one hand, such simplified environment misses the long-run effects of stabilization policies. On the other hand, it does not take into account that the optimal response to temporary shocks depends on how steady state distortions are addressed.

We conduct our analysis under two alternative policy regimes. As an interesting benchmark, we first consider the case of a Ramsey planner choosing all policy instruments (henceforth, the full Ramsey case). Then, we analyze the optimal policy mix when the monetary authority can commit, while the fiscal authority acts under discretion (henceforth, the case of fiscal discretion). The assumption is motivated by the different institutional environments in which monetary and fiscal policies are conducted. In many countries, advanced and emerging, monetary and fiscal policies are decided by separate authorities. Institutional reform in the 1980s and 1990s has made central banks independent from the treasuries, sometime also giving them an explicit mandate to achieve specific goals in terms of inflation and/or economic activity. Treasuries are not subject to explicit mandates, but they follow the electoral cycle and may be replaced at the next election. Hence, the assumption of commitment in fiscal policy seems at odds with reality. As an empirically relevant alternative to the full Ramsey benchmark, we model the case of monetary commitment and fiscal discretion as a non-cooperative game.

When the monetary authority commits to a Taylor-type interest-rate rule, we find that the unique steady-state level of debt increases with the degree of price stickiness.

Our findings contribute to the literature in several respects. First, in the full Ramsey case inflation stabilization generates permanent changes in employment, because of debt dynamics. The mechanism is straightforward. In line with the known results of the monetary policy literature, the price mark-up fluctuations induced by nominal rigidities
are responsible for making a strong case in favor of price stability. Hence, the stock of
debt is permanently affected by temporary shocks, because the market of government
bonds cannot be completed by resorting to inflation fluctuations. On the other hand, the
interest rate changes required to keep inflation stable drive debt dynamics, which in turn
permanently feeds back to employment through the required tax adjustment. For example,
consider an economy that is endowed with positive public debt and that is in an initial
state distorted by positive tax rates and imperfectly competitive markets. A positive
technology shock triggers a fall in the nominal interest rate that permanently reduces
the outstanding public debt. Facing a lower expenditure for interest rate payments, the
government can reduce tax-rates and increase hours worked. As a consequence, hours are
not fully stabilized after a technology shock as it is the case in the baseline model. The
resulting fluctuations are small, but permanent. Therefore, as long as the central bank
stabilizes inflation, a sufficiently long sequence of temporary productivity improvements
significantly and permanently increases employment. In general, size and magnitude of
the effect depend on size and magnitude of the initial level of debt. For instance, if the
government starts with assets, the mechanism outlined above is inverted and a temporary
and positive productivity shock permanently reduces employment. These results do not
obtain when prices are flexible, because inflation fluctuations are not costly and can thus
be used to complete financial markets and make debt stationary.

Second, temporary shocks do not permanently affect employment under fiscal discre-
tion. However, a link between stabilization policy and steady state distortions still exists,
even though it goes the other way. In particular, we find that optimal inflation stabiliza-
tion generates employment fluctuations that are more abrupt on impact if the economy
is distorted on average. Once again, debt accumulation and policy interaction are re-
ponsible for this result. It is well-known that debt is stationary if the fiscal authority
cannot commit even though debt is non-contingent. For instance, Debortoli and Nunes
(2012) argue that Markov perfect fiscal policy implies zero debt as a steady state. In
fact, only in a situation of balanced budget the discretionary policy maker does not have
any incentive to generate unexpected changes of the real interest rate in her own favor.
We additionally find that if prices are sticky and markets are imperfectly competitive,
the steady state level of debt is negative. This is because in our model a discretionary
fiscal policy maker also has the incentive to generate surprise inflation, so as to reduce
steady state distortions. Only for a negative level of debt indeed the gains of unexpected
deflation in terms of higher real interest rates are compensated by the gains of unexpected
inflation in terms of higher employment. It follows that the central bank’s response to
shocks does not permanently affect the employment level since debt and tax rates are
stationary and then unaffected in the long-run by changes in the nominal interest rate.
However the stabilization of inflation pursued by the central bank coupled with station-
arity of debt links the volatility of the nominal interest rate to the volatility of tax rates.
The fiscal authority indeed has to match changes in tax rates with changes in interest rate
payments so as to keep debt constant in the long-run. This fact results in higher volatility,
on impact, of hours worked. The result is quite counterintuitive: fiscal discretion and the
lack of control of the nominal interest rate on the part of the fiscal authority do not imply
excessively high or volatile debt. On the contrary, they lead to excessively high volatility
of taxes and hours at the time the shock hits. On the other hand, the impact of the shock
is temporary so that taxes and hours are less volatile at lower frequencies. In other words,
fiscal discretion, as opposed to the full Ramsey case, shifts the volatility of employment
from the long-run to the short-run.

Finally, we find that exogenous fluctuations in wage mark-ups do not imply high
volatility of inflation, irrespective of whether monetary and fiscal policy are coordinated. Also, the allocation does not differ significantly under the two policy regimes. Finally, under full Ramsey, transitory changes of the interest rate have a negligible impact on employment in the long-run. All those results can be explained by the fact that policy makers have enough instruments to cope with the shock, since tax rates can be used to counterbalance changes in mark-ups.

2 Relevant literature

Optimal monetary and fiscal policy are commonly investigated under the assumption that a single authority chooses all policy instruments. Lucas and Stokey (1983) study the choice of optimal monetary and fiscal policy under commitment in real model with perfect competition in goods and factor markets. The government faces an exogenous and stochastic process of public spending that can be financed by levying distortionary income taxes and/or issuing state-contingent bonds. In this environment the income tax and public debt inherit the dynamic properties of the exogenous stochastic disturbances. Chari, Christiano and Kehoe (1991) extend the model by Lucas and Stokey (1983) to a monetary economy where government issues nominal non-state-contingent debt. Optimal fiscal policy implies that the tax rate on labor remains essentially constant. On the other hand, optimal monetary policy requires the government to inflate in response to bad shocks and deflate in response to good shocks. Hence, inflation makes nominal debt state-contingent in real terms and it is highly volatile. In the model by Chari et al. (1991) prices are flexible and inflation is not costly. Schmitt-Grohé and Uribe (2004) analyze a model with imperfect competition and sticky product prices; the fiscal side of the model assumes that the government cannot rely on lump-sum taxes but can only use distortionary income taxes and that it can only issue nominal one-period non-state-contingent bonds. The key finding of this paper is that optimal monetary and fiscal policy under commitment implies low inflation volatility while fiscal variables display near-unit root behavior. This finding arises even for a very small degree of price stickiness. Aiyagari, Marcet, Sargent and Seppala (2002) study optimal policies in a real model with commitment and find that the dynamic behavior of fiscal variables depends on the ability of the government to issue state-contingent real debt.

Recent work has focused on discretionary policymaking while retaining the assumption of a single policy authority. Debortoli and Nunes (2012) extend the model of Lucas and Stokey (1983) with endogenous public expenditure and study optimal policymaking in the absence of commitment. The key result of this paper is that lack of commitment does not lead to debt accumulation. In fact, steady-state debt is no longer indeterminate under discretion and debt converges back to such level. Interestingly, the steady-state of the economy involves no debt accumulation at all. Niemann, Pichler and Sorger (2010) study discretionary monetary and fiscal policies in a monetary model with nominal non-state-contingent debt. The steady-state levels of debt and inflation depend on transaction costs associated with private consumption and inflation volatility falls when prices are sticky. Campbell and Wren-Lewis (2008) evaluate the welfare consequences of shocks at the efficient steady state and find substantially larger welfare costs of discretion relative to commitment. The common result among these works is that, when the monetary and fiscal authorities share the ability to commit or the lack of it, monetary policy bears the brunt of burden of stabilization while fiscal policy is in some sense neutral. In particular, government spending does not play a role in macroeconomic stabilization, as emphasized

Unlike these contributions, our paper investigates the interaction between committed monetary policy and discretionary fiscal policies. The literature on the strategic interaction between monetary and fiscal policy has typically assumed a rich game-theoretic environment in a simple model. Dixit and Lambertini (2003) explicitly model monetary and fiscal policies as a non-cooperative game between two independent authorities. The central bank being can commit while the fiscal authority acts under discretion. Their central bank is not benevolent but conservative as in Rogoff (1985) and the model is static. Adam and Billi (2010) consider independent monetary and fiscal authorities acting under discretion to analyze the desirability of making the central bank conservative to eliminate the steady-state inflation bias. When they consider committed monetary policy, their analysis is limited to the steady state.

Gnocchi (2012) analyzes the interaction of monetary commitment and fiscal discretion in a dynamic monetary model with sticky prices. In that paper the asymmetry in the policy regime leads to use of the fiscal instrument that imposes a negative externality on the central bank. In particular, the volatility of the government expenditure is inefficiently high and greater than the one implied by the solution of a Ramsey planner. This is because the government, though sharing the same objective function of the central bank, does not agree on the optimal stabilization plan implemented by the monetary authority.

The existing literature on monetary-fiscal and commitment-discretion interactions abstracts from distortionary taxation and nominal debt. This is the novelty contribution of our paper. Extending the analysis in this direction provides insight on whether discretionary fiscal policy generates inefficient volatility of debt and/or public spending and therefore provide a rationale for budget rules.

3 The Model

We follow Schmitt-Grohé and Uribe (2004) and consider a New-Keynesian model with imperfectly competitive goods markets and sticky prices. A closed production economy is populated by a continuum of monopolistically competitive producers and an infinitely lived representative household deriving utility from consumption goods, government expenditure and leisure. Each firm produces a differentiated good by using as input the labor services supplied by the household in a perfectly competitive labor market. Prices of consumption goods are assumed to be sticky à la Rotemberg (1982).

In order to keep the state-space dimension tractable, we depart from Calvo (1983) pricing, which introduces price dispersion as an additional state variable. Since Schmitt-Grohé and Uribe (2004), this is a widespread modelling choice in the literature when solving for optimal policy problems without resorting to the linear-quadratic approach.
on labor income and issuing one-period nominal non-state contingent government debt. By no arbitrage, the interest rate on bonds has to equalize the monetary policy rate in equilibrium.\footnote{We abstract indeed from default on the part of both the fiscal authority and private agents, so that the only bond traded at equilibrium is risk-free. In addition, we assume that the budget constraint holds at any history, because the paper does not focus on the fiscal theory of the price level.} Finally, we assume that the central bank and the fiscal authority are fully independent, i.e. they do not act cooperatively and they do not share a budget constraint.

This section briefly describes our economy and defines competitive equilibria.

### 3.1 Households

The representative household has preferences defined over private consumption, $C_t$, public expenditure, $G_t$, and labor services, $N_t$, according to the following utility function:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[ (1 - \chi) \ln C_t + \chi \ln G_t - \frac{(N_t)^{1+\phi}}{1+\phi} \right],$$  \hspace{1cm} (1)$$

where $\beta \in (0, 1)$ is the subjective discount factor, $E_0$ denotes expectations conditional on the information available at time 0, $\phi$ is the inverse elasticity of labor supply and $\chi$ measures the weight of public spending relatively to private consumption. Also, as we show below, $\chi$ determines the share of government expenditure over GDP, computed at the non-stochastic steady state of the Pareto efficient equilibrium. $C_t$ is a CES aggregator of the quantity consumed $C_t(j)$ of any of the infinitely many varieties $j \in [0, 1]$ and it is defined as:

$$C_t = \left[ \int_0^1 C_t(j)^{\eta_p - 1} \frac{1}{\eta_p} dj \right]^{\frac{1}{\eta_p}}.$$  \hspace{1cm} (2)$$

$\eta_p > 1$ is the elasticity of substitution between varieties. In each period $t \geq 0$ and under all contingencies the household faces the following budget constraint:

$$\int_0^1 P_t(j)C_t(j) \, dj + \frac{B_t}{1 + i_t} = W_t N_t (1 - \tau_t) + B_{t-1} + T_t.$$  \hspace{1cm} (3)$$

$P_t(j)$ stands for the price of variety $j$, $W_t N_t (1 - \tau_t)$ is after-tax nominal labor income and $T_t$ represents nominal profits rebated to the household by firms. The household can purchase nominal government debt $B_t$ at the price $1/(1 + i_t)$, where $i_t$ is the nominal interest rate. The nominal debt $B_t$ pays one unit in nominal terms in period $t + 1$. To prevent Ponzi games, the following condition is assumed to hold at all dates and under all contingencies

$$\lim_{T \to \infty} E_t \left\{ \prod_{k=0}^{T} (1 + i_{t+k})^{-1} B_{t+T} \right\} \geq 0.$$  \hspace{1cm} (4)$$

Given prices, policies and transfers $\{P_t(j), W_t, i_t, G_t, \tau_t, T_t\}_{t \geq 0}$ and the initial condition $B_{-1}$, the household chooses the set of processes $\{C_t(j), C_t, N_t, B_t\}_{t \geq 0}$, so as to maximize (1) subject to (2)-(4). After defining the aggregate price level\footnote{The price index has the property that the minimum cost of a consumption bundle $C_t$ is $P_t C_t$.} as:
\[ P_t = \left[ \int_0^1 P_t(j)^{1-\eta_p} dj \right]^{1-\eta_p}, \]  

as well as real debt, \( b_t \equiv B_t/P_t \), the real wage \( w_t \equiv W_t/P_t \) and the gross inflation rate, \( \pi_t \equiv P_t/P_{t-1} \), optimality is characterized by the standard first order conditions:

\[ C_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\eta_p} C_t, \]  
\[ \beta E_t \left\{ \frac{C_t(1+i_t)}{C_{t+1}\pi_{t+1}} \right\} = 1, \]  
\[ \frac{N_t}{1-\chi} = w_t(1-\tau_t) \]

(8)

together with transversality:

\[ \lim_{T \to \infty} E_t \left\{ \beta^T \frac{b_t+T}{C_t+T} \right\} = 0. \]  

(9)

Equation (8) shows that the labor income tax drives a wedge between the marginal rate of substitution between leisure and consumption and the real wage.

### 3.2 Firms

There are infinitely many firms indexed by \( j \) on the unit interval \([0, 1]\) and each of them produces a differentiated variety with a constant return to scale technology

\[ Y_t(j) = z_t N_t(j), \]  

(10)

where productivity \( z_t \) is identical across firms and \( N_t(j) \) denotes the quantity of labor hired by firm \( j \) in period \( t \). Following Rotemberg (1982), we assume that firms face quadratic price adjustment costs:

\[ \frac{\gamma}{2} \left( \frac{P_t(j)}{P_{t-1}(j)} - 1 \right)^2 \]  

(11)

expressed in the units of the consumption good defined in (2) and \( \gamma \geq 0 \). The benchmark of flexible prices can easily be recovered by setting the parameter \( \gamma = 0 \). Nominal profits read as:

\[ E_t \left\{ \sum_{s=0}^{\infty} Q_{t,t+s} \left[ P_{t+s}(j) Y_{t+s}(j) - W_{t+s} N_{t+s}(j) - P_{t+s} \frac{\gamma}{2} \left( \frac{P_{t+s}(j)}{P_{t+s-1}(j)} - 1 \right)^2 \right] \right\}, \]  

(12)

where \( Q_{t,t+s} \) is the discount factor in period \( t \) for nominal profits \( s \) periods ahead. Assuming that firms discount at the same rate as households implies

\[ Q_{t,t+s} = \beta^s \frac{C_t}{C_{t+s}\pi_{t+s}}. \]  

(13)

Each firm faces the following demand function:
\[ Y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\eta p} Y_t^d, \]  

where \( Y_t^d \) is aggregate demand and it is taken as given by any firm \( f \). Firms choose processes \( \{ P_t(j), N_t(j), Y_t(j) \}_{t \geq 0} \) so as to maximize (12) subject to (10) and (14), taking as given aggregate prices and quantities \( \{ P_t, W_t, Y_t^d \}_{t \geq 0} \). Let the real marginal cost be denoted by \( mc_t = w_t/z_t \). Then, at a symmetric equilibrium where \( P_t(j) = P_t \) for all \( j \in [0, 1] \), profit maximization and the definition of the discount factor imply:

\[ \pi_t(\pi_t - 1) = \beta E_t \left[ \frac{C_t}{C_{t+1}} \left( \frac{\pi_{t+1}}{\pi_t} - 1 \right) \right] + \frac{\eta p z_t N_t}{\gamma} \left( mc_t - \frac{\eta p - 1}{\eta p} \right). \]  

(15) is the standard Phillips curve according to which current inflation depends positively on future inflation and current marginal cost.

### 3.3 Policymakers

In the economy there are two benevolent policy makers. The monetary authority is responsible for setting the nominal interest rate \( i_t \). The fiscal authority provides the public good \( G_t \) that is obtained by buying quantities \( G_t(j) \) for any \( j \in [0, 1] \) and aggregating them according to:

\[ G_t = \left[ \int_0^1 G_t(j) \frac{\eta p - 1}{\eta p} \, dj \right] \frac{\eta p}{\eta p - 1}, \]  

(16)

so that total government expenditure in nominal terms is \( P_t G_t \) and the public demand of any variety is:

\[ G_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\eta p} G_t. \]  

(17)

Expenditures are financed by levying a distortive labor income tax \( \tau_t \) or by issuing one-period, risk-free, non-state contingent nominal bonds \( B_t \). Hence, the budget constraint of the government is:

\[ \frac{B_t}{1 + i_t} + \tau_t W_t N_t = B_{t-1} + G_t P_t. \]  

(18)

Therefore, the central bank and the fiscal authority determine the sequence \( \{ i_t, G_t, \tau_t \}_{t \geq 0} \) that, at the equilibrium prices, uniquely determines the sequence \( \{ B_t \}_{t \geq 0} \) via (18). For what follows, the government budget constraint can be rewritten in real terms

\[ \frac{b_t}{1 + i_t} + \tau_t mc_t z_t N_t = \frac{b_{t-1}}{\pi_t} + G_t, \]  

(19)

after substituting for \( w_t \) from the expression for the real marginal cost.

### 3.4 Competitive equilibrium

At a symmetric equilibrium where \( P_t(j) = P_t \) for all \( j \in [0, 1] \), \( Y_t(j) = Y_t^d \) and the feasibility constraint is:

\[ z_t N_t = C_t + G_t + \frac{\gamma}{2} (\pi_t - 1)^2, \]  

(20)
where

\[ N_t = \int_0^1 N_t(j) \, dj \]  \quad (21)

is the aggregate labor input. Productivity is stochastic and evolves according to the following process

\[ \ln z_t = \rho \ln z_{t-1} + \epsilon_t, \]  \quad (22)

where \( \epsilon_t \) is an i.i.d. shock and \( \rho \) is the auto-regressive coefficient.

We define the notion of competitive equilibrium as in Barro (1979) and Lucas and Stokey (1983), where decisions of the private sector and policies are described by collections of rules mapping the history of exogenous events into outcomes, given the initial state. To simplify notation we stuck private decisions and policies into vectors

\[ x_t = \{C_t, N_t, b_t, mc_t, \pi_t\} \]  \text{ and } \[ p_t = \{i_t, G_t, \tau_t\}, \]

respectively. Let \( s_t = (z_0, ..., z_t) \) be the history of exogenous events. Given a particular history \( s^r \) and the endogenous state \( b_{t-1} \), \( x_r(s^r|s_t, b_{t-1}) \) and \( p_r(s^r|s^t, b_{t-1}) \) denote the rules describing current and future decisions for any possible history \( s^r, r \geq t, t > 0 \). Finally, we can define a continuation competitive equilibrium as a set of sequences\footnote{To simplify notation we suppress functional arguments.} \( A_t = \{x_r, p_r\}_{r \geq t} \) satisfying equations (7)-(9), (15) and (19)-(20) for any \( s^r \). Obviously, a competitive equilibrium \( A_0 \) is simply a continuation competitive equilibrium starting at \( s^0 \), given \( b_{-1} \).

3.5 The Pareto-efficient allocation

The Pareto-efficient allocation solves the problem of maximizing utility (1) subject to equations (2), (10), (16), (21) and the resource constraint \( Y_t(j) = C_t(j) + G_t(j) \) for any \( j \). It can be showed that Pareto-efficiency requires \( C_t(j) = C_t, Y_t(j) = Y_t, G_t(j) = G_t \) and \( N_t(j) = N_t \). Moreover, the marginal rate of substitution between leisure and private consumption and between leisure and public consumption must be equal to the corresponding marginal rate of transformation. This implies

\[ z_t = N_t^\phi \frac{C_t}{1 - \chi} = N_t^\phi \frac{G_t}{\chi}. \]  \quad (23)

The optimality conditions yield the efficient allocation

\[ \begin{align*}
\mathcal{N}_t &= 1; \\
\mathcal{Y}_t &= z_t; \\
\mathcal{C}_t &= (1 - \chi) z_t; \\
\mathcal{G}_t &= \chi z_t.
\end{align*} \quad (24)\]

Under Pareto efficiency hours worked are constant, while consumption, government expenditure and output move proportionally to productivity.

4 Monetary and fiscal policy coordination

We take as benchmark the case of coordination, where a single authority chooses monetary and fiscal policy instruments under commitment. We follow the classic Ramsey (1927) approach and we define the optimal policy as a state-contingent plan. We refer to this case as Full Ramsey (FR). As much of the existing literature, we also assume that the initial inflation rate is given so as to avoid the equilibrium where the Ramsey planner inflates...
Table 1: Benchmark Calibration

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight of G in utility</td>
<td>$\chi$</td>
<td>0.15</td>
</tr>
<tr>
<td>Weight of C in utility</td>
<td>$1 - \chi$</td>
<td>0.85</td>
</tr>
<tr>
<td>Elast. subst. goods</td>
<td>$\eta^p$</td>
<td>11</td>
</tr>
<tr>
<td>Price stickiness</td>
<td>$\gamma$</td>
<td>20</td>
</tr>
<tr>
<td>Serial corr. tech.</td>
<td>$\rho_z$</td>
<td>0</td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>Frisch elasticity</td>
<td>$\varphi^{-1}$</td>
<td>1</td>
</tr>
</tbody>
</table>

away the nominal wealth of the household by choosing an infinite price level.\textsuperscript{5} Therefore, we can define the FR equilibrium as a competitive equilibrium $A_0$ that maximizes $U_0$, given the initial conditions $b_{-1}$ and $\pi_0$. The Lagrangian and the first-order conditions associated to the FR problem are reported in Appendix A.

The Ramsey solution has been analyzed before and section ?? summarized the findings of this literature. An aspect that has not received much attention is that the Ramsey planner’s incentive to undo the distortions in the economy varies with steady-state level of debt. We turn to this issue next.

4.1 Optimal policies and the level of debt

Our benchmark economy features two distortions: a) imperfect competition in the goods market; b) price-adjustment costs. The deep parameters of the model are set according to Table 1. The weight $\chi$ in the utility function has been chosen to roughly match U.S. post-war government spending-to-GDP ratio. We set the serial correlation of the technological shock equal to zero to help us understand the mechanisms at play. The log-linearized Philips curve (15) is as follow:

$$\hat{\pi}_t = \frac{\pi - 1}{2\pi - 1} \beta(\hat{C}_t - \hat{C}_{t-1}) + \beta \pi_{t+1} + \frac{Y(\eta^p - 1)}{\gamma\pi(2\pi - 1)} \hat{mc}_t,$$

where a circumflex denotes log-deviations from steady state and variables without a time subscript denote steady-state values. The effect of variations in the marginal cost on current inflation depends on the parameters $\gamma$ and $\eta^p$ but also on steady-state output and inflation, where the former depends on the initial level of government debt. We set the parameter $\gamma$ that measures the degree of price stickiness equal to 20 for our benchmark calibration. Insert here comment on the implied degree of price stickiness. Appendix B performs robustness checks for the parameters $\eta^p$ and $\gamma$.

We consider three steady-state levels of government debt.\textsuperscript{6} The first steady state is the efficient equilibrium of our model. This is the allocation where a labor subsidy completely eliminates the monopolistic distortion stemming from imperfect competition in the goods market.\textsuperscript{5} This assumption is not necessary in the presence of price adjustment costs, as pointed out by Schmitt-Grohé and Uribe (2004). Nevertheless we retain this assumption to ease comparison of our results with those in the literature.

\textsuperscript{6}More precisely, we choose the steady-state level of debt knowing that there exists an initial condition that supports it and we abstract from the transition from such initial condition to the chosen steady state.

\textsuperscript{5}
Table 2: Steady State

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Efficient</th>
<th>$b/Y = 1$</th>
<th>$b/Y = -1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>$C$</td>
<td>0.85</td>
<td>0.74</td>
<td>0.75</td>
</tr>
<tr>
<td>Government expenditure</td>
<td>$G$</td>
<td>0.15</td>
<td>0.13</td>
<td>0.13</td>
</tr>
<tr>
<td>Hours worked</td>
<td>$N$</td>
<td>1</td>
<td>0.87</td>
<td>0.88</td>
</tr>
<tr>
<td>Real debt</td>
<td>$b$</td>
<td>-24.0909</td>
<td>0.87</td>
<td>-0.88</td>
</tr>
<tr>
<td>Income tax</td>
<td>$\tau$</td>
<td>-0.1</td>
<td>0.1735</td>
<td>0.1518</td>
</tr>
<tr>
<td>Gross inflation</td>
<td>$\pi$</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

market. Since lump-sum taxes do not exist in our model, the labor subsidy as well as the provision of the public good must be financed with interest receipts on government assets. This implies that, at the efficient steady state,

$$b^{eff} = \frac{1/\eta^p - \chi}{1 - \beta}, \quad \tau^{eff} = -\frac{1}{\eta^p - 1}, \quad C^{eff} = 1 - \chi, \quad G^{eff} = \chi, \quad N^{eff} = Y^{eff} = 1.$$  

The lagrangian multipliers on the government budget constraint $\lambda^s$, on the Euler equation $\lambda^b$, and on the Phillips curve $\lambda^p$ are equal to zero at the efficient steady state. The values of the macroeconomic variables of interest are reported in the third column of Table 2. In words, public assets must be 24 times GDP for the interest income to be sufficiently high to finance subsidies and government spending. Since public assets are private liabilities in our model, the assumption of commitment to repay on the side of private agents may appear unrealistic with such high level of indebtedness. But we consider the efficient steady state as a theoretical benchmark and maintain the assumption that all debts are repaid – private or public.

The second steady state features a positive level of government debt that, without loss of generality, we set equal to GDP. In the third steady state the government is a creditor and public assets are equal to GDP. These two steady states are summarized in the fourth and fifth column of Table 2. In the economy with positive public debt the tax rate is 17.35% at the steady state and hours worked are well below their efficient level. As a result, the steady state of the economy is distorted and output is below its efficient level. The economy with public credit (negative public debt) has a steady-state tax rate of 15.18%, which implies higher hours and output relative to the economy with public debt.

To illustrate how steady-state debt affects optimal policies, we analyze the dynamic responses of the economy starting at the three steady states specified in Table 2. Consider first the case where prices are fully flexible, namely $\gamma = 0$. Figure 1 presents the impulse responses of our key variables to a positive technological shock. We fix the size of the shock to the typical standard deviation considered in the business cycle literature, 0.0071. Then we normalize consumption, government expenditure, hours worked and output with respect to the shock and we report them in percentage deviations from the steady state. Hence, a one percent increase in a given variable means that the variable increases as much as productivity. Inflation and the nominal interest rate are not normalized, but rather expressed in percentage points and reported in deviation from the steady state. For instance, a 0.01 denotes that the variable in question is one hundredth of a percentage
point above its steady-state level. Finally, tax rates and real debt are not normalized either and they are reported in percentage deviation from their steady-state values.

Under flexible prices ($\gamma = 0$), the impulse responses of the economies starting at the three different initial conditions are identical. A technological improvement raises private and public spending as much as output. Inflation and the nominal interest rate fall on impact while the tax rate, hours and public debt are completely stabilized. Since prices are flexible, inflation adjusts so as to make real government debt/assets state-contingent. Inflation falls, thereby raising real government debt repayment, and the nominal interest rate falls as well, thereby raising the price of newly issued bonds and raising private consumption. The effect on the government budget constraint of lower inflation dominates that of lower nominal interest rate and generates additional revenues when the government is a creditor but additional outlays when the government is a debtor. Thanks to higher productivity and wages, subsidies increase in the economy at the efficient steady state. Increased government outlays, as measured by subsidies plus government spending, are financed by the increase in the real public asset revenues and, as a result, government debt is completely stabilized. The other economies experience an increase in tax revenues that finances higher government spending and real debt repayment. For these economies as well debt is completely stabilized. As in Lucas and Stokey (1983) and Chari et al. (1991), fiscal variables inherit the dynamic properties of the shock.

The presence of sticky prices ($\gamma = 20$) alters these findings. The impulse responses to a positive technological shock are reported in Figure 2; the magnitude of the shock is the same as in Figure 1. Consider first the dynamic responses when $b/Y = 1$ and $b/Y = -1$. Since changing prices is costly, inflation is stabilized. Relative to the case...
with flexible prices, real debt repayment is also stabilized, which leads to lower outlays in the presence of government debt \( (b > 0) \) but lower revenues in the case of government assets \( (b < 0) \). As in the case with flexible prices, labor income taxes are kept unchanged so that hours worked are stabilized; the nominal interest rate is reduced so as to raise private consumption. Increased tax revenues due to higher wages finance an increase in public spending. The response of output and consumption, both private and public, is Pareto-efficient. Inflation stabilization has two consequences. First, it generates a budget surplus when the government is a debtor (and a deficit when government is a creditor) in the short run in response to a technological improvement. This is the reduction in \( b \) for \( b/Y = 1 \) (and the increase in \( b \) for \( b/Y = -1 \)) shown in Figure 2. Second, it induces a unit root in public debt that turns short-run budget imbalances in long-run debt changes.

The dynamic response of the economy around the efficient steady state stands by itself. Neither inflation nor hours are stabilized; the response of output and consumption, both private and public, is unlike that seen with flexible prices. Intuitively, public assets are so large that the short-run response to a shock is constrained by its long-run consequences. Lowering the nominal interest rate on such large amount of private debt causes prohibitive costs for the government. As a result, the Ramsey planner reduces the nominal interest rate less, which in turn stimulates private consumption less, than under flexible prices. The Ramsey planner reduces the labor income subsidy \( \tau \) in order to reduce outlays. Although changing prices is costly, inflation is slightly reduced in order to generate revenues. Notice that government spending increases more than under flexible prices as the Ramsey planner uses \( G \) to stabilize aggregate demand and compensate for lower private consumption.
5 The interaction of monetary commitment and fiscal discretion

In this section we model policymaking as a non-cooperative game where monetary and fiscal policies are conducted by two separate and independent authorities. We assume that both policymakers are benevolent and maximize social welfare, but only the monetary authority can credibly commit to future policies. In contrast, the fiscal authority cannot do so and therefore acts under discretion. We are interested in analyzing time-consistent fiscal policy under a variety of monetary arrangements, within the class of monetary commitment. Hence, we first describe the game in a general form, defining timing and strategy space. We do so by following the same formalism as in Chari and Kehoe (1990) and Atkeson, Chari and Kehoe (2010). Then, we consider alternative policy regimes by varying the restrictions we impose on monetary strategies and computing the resulting equilibrium.

5.1 The policy game

A formal description of the game allows us to be transparent about the assumptions we make on the strategies available to the monetary and fiscal authorities. Also, it allows to establish a clear connection between outcomes of the game and competitive equilibria.

We focus on the strategic interaction between policymakers and regard households and firms as non-strategic.\(^7\) The events of the game unfold according to the following time-line. In period \(t = 0\), at a stage that one may consider as constitutional, the central bank commits once and for all to a rule, say \(\sigma_m\), contingent on all information available when \(i_t\) will have to be set. Then, in every period \(t \geq 0\): a) shocks occur and they are perfectly observed by all agents and authorities; b) the fiscal authority chooses its fiscal tools; c) the monetary authority implements the plan it committed to at the constitutional stage; d) economic variables realize. Accordingly, the history of the game can be defined as \(h_t = (q_t, h_{t-1})\), \(t > 0\), where the events occurring in each period are represented chronologically by \(q_t = (z_t, G_t, \tau_t, i_t, x_t)\) and \(h_0 = (q_0, b_{-1})\). Our timing assumption implies that the central bank leads both the fiscal policymaker and private agents. The fiscal policymaker only leads private agents within each period. However, as it will become evident below, it is convenient to think of the fiscal policymaker as a sequence of authorities with identical preferences, each one leading her future selves.\(^8\) Now we can define the strategies for the authorities. The fiscal authority faces histories \(h_{t,f} = (h_{t-1}, z_t)\) so that her strategy is \(\sigma_f = \{G_t(h_{t,f}), \tau_t(h_{t,f})\}_{t \geq 0}\). Similarly, the monetary policy instrument is contingent on \(h_{t,m} = (h_{t-1}, z_t, G_t, \tau_t)\). Therefore, the strategy of the central bank is \(\sigma_m = \{i_t(h_{t,m})\}_{t \geq 0}\). Finally, the information available to private agents in each period can be represented by \(h_{t,x} = (h_{t-1}, s_t, G_t, \tau_t, i_t)\). Economic variables realize according to \(\sigma_x = \{x_t\}_{t \geq 0}\), where \(x_t\) are the decision rules defined in section 3.4. For any strategy, it is convenient to define its continuation from a given history. For instance, consider fiscal strategy \(\sigma_f\). Hence, its continuation from \(h_{t,f}\) is \(\sigma^t_f = \{G_t(h_{r,f}), \tau_t(h_{r,f})\}_{r \geq t}\). Starting from any history of the game \(h_{t-1}\), a strategy profile \(\sigma = (\sigma_f, \sigma_m, \sigma_x)\) naturally induces

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\(^7\) In contrast, Atkeson et al. (2010) study off-equilibrium behavior of price setters so as to find monetary policy strategies uniquely implementing any competitive equilibrium.

\(^8\) If one restricts to the case of monetary commitment, inverting the order of moves within each period would not change our results. In fact, the central bank could still condition the nominal interest rate on the history of fiscal instruments. See Gnocchi (2013) for further discussion.
continuation outcomes. As an example, we show a recursion for a generic time \( t \). Consider history \( h_{t-1} \). Then, the following events happen: a) \( z_t \) realizes and it is observed by all agents; b) \( h_{t,f} \) becomes known and \( \sigma_f^{\ast} \) generates outcomes \( G_t \) and \( \tau_t; c) \) \( h_{t,m} \) is observed by the central bank and \( i_t \) is set according to \( \sigma_{i,m}^{\ast} \); d) economic variables finally realize as prescribed by \( \sigma_{x}^{\ast} \) and \( h_t \) is thus determined. Similarly, for any possible future history of exogenous events \( s', r > t \), future outcomes can be generated and we can collect them all in \( B_t(s' | h_{t-1} ; \sigma) = \{ x_r, i_r, G_r, \tau_r \}_{r \geq t} \), which denotes the continuation of the game from a particular history \( h_{t-1} \). Since we regard the private sector as non-strategic, we disregard its behavior off-equilibrium. Accordingly, we require that the conditions for a competitive equilibrium (7)-(9), (15) and (19)-(20) are satisfied for all histories. In other words, we assume that \( \sigma_* \) induces outcomes \( B_t \) that are continuation competitive equilibria for any \( h_{t,x} \). In contrast, according to our general definition of the game, we let the central bank free to condition the nominal interest rate on any history \( h_{t,m} \), irrespective of whether the history is on the equilibrium path or not. If no further restrictions on the monetary strategies are imposed, the central bank might use off-equilibrium threats. In section 5.2 we rule out off-equilibrium threats while we allow for them in section 5.3, where we also discuss extensively their implications. Finally, we assume that the fiscal strategy only depends on the history of exogenous events \( s' \) and on the inherited level of debt \( b_{t-1} \). Also, we restrict functions \( G_r(h_{r,f}) \) and \( \tau_r(h_{r,f}) \) to be differentiable. These assumptions are standard when one wants to restrict to Markov-perfect equilibria. Now, we can define optimality of policies as in Atkeson et al. (2010). A fiscal policy strategy \( \sigma_f^* \) is optimal if it maximizes \( U_t \) for any \( h_{t,f} \) and for any monetary strategy \( \sigma_m \), given continuation (\( \sigma_f^{\ast} \)). Intuitively, the current fiscal authority chooses \( G_t \) and \( \tau_t \) in order to maximize utility and by taking into account that future policies will also be chosen optimally by her future selves. A monetary policy strategy \( \sigma_m^* \) is optimal if it maximizes \( U_0 \), given \( \sigma_f^* \). Notice that our definition of optimality invokes perfection, both for monetary and fiscal policies. In fact, we require maximization with respect to \( \sigma_f \) for any \( \sigma_m \), rather than maximizing at the equilibrium monetary strategy \( \sigma_m^* \) as in a non-perfect Nash equilibrium.

### 5.2 Ramsey monetary policy and fiscal discretion

We start by considering the case where monetary policy is optimal according to the classic Ramsey (1927) approach, according to which the optimal plan prescribes an interest rate path that is not contingent on the fiscal authority’s actions, but only on the exogenous history. We analyze both the steady-state and the dynamic response to shocks. For the former we find that the level of debt is an outcome of strategic interaction between successive fiscal authorities. Furthermore, we show that such interaction depends on nominal price rigidities as well as on the steady-state distortion. We also show that strategic interaction makes the fiscal authority excessively concerned with debt stabilization as compared to the FR case. In fact, when the fiscal authority cannot commit, she wants to avoid suboptimal fiscal policies in the future and therefore leaves to her successor a level of debt that eliminates the incentive to manipulate the real interest rate. As an implication, taxes and hours worked are sub-optimally volatile. This fact worsens the stabilization

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9Therefore, if one fixes strategies \( \sigma \), continuations of the game map the history of exogenous events into outcomes and they can be directly compared with continuation competitive equilibria.

10See Klein, Krusell and Rios-Rull (2008) for a discussion. Debortoli and Nunes (2012) and Ellison and Rankin (2007) are additional examples on the use of the differentiability restriction.

11Commitment and discretion are indeed perfect equilibria of two different games, not different equilibrium refinements of the same game. See Chari and Kehoe (1990) for further discussion.
trade-off faced by the central bank. However, the optimal monetary response is to disregard fiscal misbehavior and pursue an aggressive inflation stabilization policy, similarly to the full Ramsey case. (we will maybe do sensitivity with respect to this, before editing the final version of this sentence.)

Our setup can easily be implemented by restricting the monetary strategy space to sequences of the form \( \sigma_m = \{ i_t(s^t, b_{-1}) \}_{t \geq 0} \). Such strategies, often labelled as open loop in dynamic game theory, are routinely assumed in the optimal policy literature. Hence, in the rest of the paper we use the terms open loop and Ramsey strategies interchangeably.\(^{12}\) We also maintain our previous assumption on the fiscal strategy space so that \( \sigma_f = \{ G_t(s^t, b_{t-1}) \}_{t \geq 0} \). It may be useful to notice that our restrictions imply a crucial difference between monetary and fiscal strategies. Government expenditure is contingent on \( b_{t-1} \). Instead, the monetary rule is conditional on \( b_{-1} \). As a consequence, the way the monetary authority interacts with the fiscal policymaker differs from the way the fiscal authority interacts at each history \( h_{t,f} \) with her future selves who play the continuation game. In particular, the fiscal authority chooses her strategy internalizing \( \sigma_m \) and our definition of optimality implies that in general any \( \sigma_m \) is associated to a different \( \sigma_f^* \). As an example, consider the case where the fiscal authority is initially endowed with a positive net asset position vis à vis the private sector and suppose that the central bank reduces the nominal interest rate whenever productivity increases. Hence, following a technology shock, the fiscal authority might have the incentive to finance expenditure by issuing relatively more debt and resorting to taxation relatively less, as compared to when the central bank does not respond to productivity. However, the central bank shapes fiscal incentives only indirectly, because her instrument is not contingent on fiscal variables. In contrast, the way fiscal policy behaves in each period directly affects her own future equilibrium choices through the state variable \( b_t \). Again as an example, suppose that it is optimal to increase taxes whenever debt increases. This fact implies that the current fiscal policymaker can directly manipulate the decision of her successor by issuing debt. If the fiscal authority has actually some interest in doing so, non-trivial strategic interaction may arise between successive fiscal policymakers. Hence, the presence of debt introduces an additional layer of interaction that is absent when balanced budget is assumed.

In the remainder of the section, we first define the equilibrium under our strategy restrictions. Then, after briefly explaining how we compute it, we illustrate our main results.

We say that a strategy profile \( \sigma^* \) is an equilibrium with Ramsey monetary policy and fiscal discretion (for short, RD henceforth) if monetary and fiscal policies are optimal in the sense defined above. We solve for the RD equilibrium in two steps by backward induction. First, for a given monetary strategy, we characterize outcomes that are optimal for the fiscal policy maker by adopting a primal approach. Hence, we decide on fiscal variables as well as on private sector choices, subject to the constraint that they must be a continuation competitive equilibrium. Then, we optimize over monetary strategies. Formally, the fiscal problem can be recursively written as\(^{13}\)

\[
W^f_t = \max_{C_t, N_t, b_t, m_t, \pi_t, G_t} \left\{ (1 - \chi) \ln C_t + \chi \ln G_t - \frac{(N_t)^{1+\varphi}}{1+\varphi} + \beta E_t W^f_{t+1} \right\} \\
\text{s.t.} \\
z_t N_t - C_t - G_t - \frac{\gamma}{2} (\pi_t - 1)^2 = 0;
\]

\(^{12}\)We postpone to the following sections the comparison with alternative monetary policy regimes.

\(^{13}\)The lagrangian function and the first-order conditions associated to this maximization problem are reported in Appendix C. We slightly abuse of notation so as to save on space by replacing \( W^f(s^t, b_{t-1}, i_t) \) with \( W^f_t \).
\[
\frac{1 - \chi}{C_t(1 + i_t)} - \beta E_t \frac{1 - \chi}{C(b_t, s_{t+1}, i_{t+1})\Pi(b_t, s_{t+1}, i_{t+1})} = 0; \\
\frac{b_t}{1 + i_t} + \left( mc_t z_t - \frac{N^p C_i}{1 - \chi} \right) N_t - \frac{b_{t-1}}{\pi_t} - G_t = 0;
\]
and
\[
\beta E_t \frac{C_t \Pi(b_t, s_{t+1}, i_{t+1})(\Pi(b_t, s_{t+1}, i_{t+1}) - 1)}{C(b_t, s_{t+1}, i_{t+1})} + \frac{\eta^p}{\gamma} z_t N_t \left( mc_t - \frac{\eta^p - 1}{\eta^p} \right) - \pi_t(\pi_t - 1) = 0,
\]

taking as given \{i_r(s^r, b_{-1})\}_{r \geq t}, b_{t-1} and unknown functions \(C\) and \(\Pi\). In addition, we require (9) to hold. For convenience and without any loss of generality, we have substituted for \(\tau_t\) into (19) from the household’s optimality condition (8). The problem is recursive and one can easily show that the monetary problem is recursive as well.\(^{14}\) Then, if an equilibrium exists, there must exist a value \(W^f_t\) and time-invariant functions \(C\) and \(\Pi\) that satisfy all the conditions for a continuation competitive equilibrium and are consistent with fiscal optimality at \(h_{t+1,f}\). In other words, we make sure that \(\sigma^f\) is optimal by checking that the outcome \(G_t\) maximizes the objective function \(U_t\) at any history \(h_{t,f}\), given that continuation outcomes are optimal too. As in Klein et al. (2008), Markov-perfection is guaranteed by taking \(C\) and \(\Pi\) as given.\(^{15}\) The monetary problem is finally solved by optimizing \(U_0\) subject to the constraints that: a) the conditions for a competitive equilibrium are satisfied, i.e. equations (7), (9), (15) and (19)-(20); b) the equilibrium outcome is optimal for the fiscal policy maker and thus satisfies the first order conditions associated to the problem (26).\(^{16}\)

We now set \(z_t = 1\) for all \(t\) and analyze the non-stochastic steady-state of the RD equilibrium. There exist two non-stochastic steady-states. The first one coincides with the efficient allocation described in section 3.5, at which all constraints are slack with the exception of the resource constraint. The first-best allocation can be implemented because the fiscal authority has large positive claims against the private sector to finance government spending and labor subsidies that undo the distortion due to monopolistic competition.\(^{17}\) The second steady-state is reported in Table 3. The level of public debt is negative, which implies that the government has net positive claims relative to the private sector. Nevertheless, these claims are not large enough to finance government expenditure and a labor subsidy. In contrast, the fiscal authority must resort to positive labor income taxes that make the steady-state distorted. The level of debt is equal to\(^{18}\)

\[
b = -\frac{\gamma}{\eta^p}.
\]

With flexible prices \(\gamma = 0\), government debt is equal to zero; as \(\gamma\) becomes positive, \(b\) becomes negative. Moreover, it decreases in absolute value with \(\eta^p\), so that in a perfectly

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\(^{14}\)The monetary problem is described below. Recursivity can thus be obtained by applying the method proposed by Marcet and Marimon (1998). Notice that we are not assuming that strategies are sequences of time-invariant functions. We simply claim that equilibrium outcomes can be represented by time-invariant functions because all problems can be cast into a recursive form.

\(^{15}\)Their differentiability follows from the differentiability restriction we impose on strategies.

\(^{16}\)Intuitively, the maximization problem of the monetary authority is now augmented by four first-order conditions (46) to (49) that describe the fiscal authority’s choice and that are reported in Appendix C. A formal description of the monetary problem is also stated in Appendix C.

\(^{17}\)Niemann et al. (2010) find also two steady states in a similar model where both monetary and fiscal policies are discretionary.

\(^{18}\)This expression comes from evaluating the first-order condition for the fiscal authority (48), stated in Appendix C, at the zero net inflation steady state.
competitive economy debt tends to zero. In our monetary economy, the fiscal authority has an incentive to use inflation to achieve two goals. First, inflation or deflation affects the real cost of debt which at the steady state is given by $b(1 - \beta)/\pi$ (a cost when $b > 0$, a revenue when $b < 0$). If the government is a debtor ($b > 0$) it has an incentive to increase inflation. Vice versa, when the government is a creditor ($b < 0$). Second, if there are price rigidities, inflation affects output via the Phillips curve and the fiscal authority has an incentive to raise output as long as the steady-state is distorted. On the other hand, inflation entails some resource cost, summarized by equation (20). Furthermore, positive inflation at a particular history $h_{t,f}$ worsens the inflation-output trade-off faced by the fiscal authority at history $h_{t-1,f}$. For example, at any point in time, the fiscal authority can expand output by tolerating some inflation cost. However, the higher is expected future inflation, the higher is the current inflation rate required to achieve a given output target. Therefore, future inflation is costly for the current government. At the RD equilibrium, such cost is not internalized by the future fiscal authority who indeed cares only about $U_{t+1}$. It follows that any fiscal policymaker has the incentive to play strategically and behave in such a way that her successor internalizes the cost of future inflation. A natural way of doing so is to run a budget surplus and accumulate some assets, so that positive future inflation increases the real burden of debt for the successor. Hence, we are ready to interpret our steady-state result. Intuitively, steady-state government credit relative to the private sector needs to be large enough to generate an incentive to reduce inflation that exactly counterbalances the incentive to raise inflation for stimulating the economy. The incentive of the fiscal authority to use inflation disappears for the level of debt in (27). These incentives are captured by the interplay of the parameters $\gamma$ and $\eta^p$. When prices are flexible and $\gamma = 0$, changes in the inflation rate do not affect output in the short run, no matter how large the steady-state distortion is. Without monopolistic distortion ($\eta^p \to \infty$) the fiscal authority does not face an incentive to raise output at the steady-state, which is in fact efficient. Hence, in both cases, it is only with a zero net asset position that the fiscal authority will not deflate so that steady-state debt is zero. On the other hand, as $\gamma$ increases the inflation-output trade-off becomes more important and when $\eta^p$ falls the distortion stemming from monopolistic competition becomes larger. In both cases the incentive to stimulate the economy is stronger and, consistently with (27), the steady-state level of public credit must increase.

Even though our main focus is on the steady-state, we now turn to the analysis of the dynamics. Figure 3 reports the impulse responses of the variables following an i.i.d.

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19In this case Debortoli and Nunes (2012) consider a real model without monopolistic competition with discretionary fiscal policy and find that the steady state has zero debt. Our setting nests such outcome.
technology shock and compares them to the FR case. We shock both economies at an
initial level of debt equal to the RD steady-state, as described in Table 3. We fix the size of
the shock to the typical standard deviation considered in business cycle literature, 0.0071.
Then, we normalize consumption, government expenditure, hours worked and output with
respect to the shock and we report them in percentage deviations from the steady-state.
Hence, a one percent increase of a given variable means that the variable increases as
much as productivity. Inflation and the nominal interest rate are not normalized; they
are expressed in percentage points and reported in deviation from the steady-state. For
instance, a 0.01 denotes that the variable in question is one hundredth of a percentage
point above its steady-state value. Finally, tax rates and real debt are not normalized
either and they are reported in percentage deviations from their steady-state values. We
limit our attention to the case where $\gamma = 20$ and price adjustments are costly. The blue
starred lines in Figure 3 present the RD equilibrium; the red circled lines present the FR
equilibrium. Three are our main points. To start with, even though inflation is more
volatile in the RD regime relative to the FR, the difference is quantitatively negligible. It
is clear that the central bank is not willing to give up inflation stabilization, whatever is
the additional trade-off that fiscal discretion might induce. As an implication, the nominal
interest rate falls approximately as much as under the FR case, a fact that ceteris paribus
deteriorates public accounts because the fiscal authority is initially endowed with positive
assets. Second, fiscal discretion worsens the trade-off between stabilizing inflation and real
activity, as it becomes clear by looking at the response of hours worked. Since Debortoli
and Nunes (2012) it is known that the economy is stationary under fiscal discretion due
to strategic interaction between successive policymakers. We extend their analysis to
a stochastic model with committed monetary policy and show that such interaction is
harmful for business cycle stabilization. Should the current fiscal authority decide not to
raise taxes, the future fiscal policymakers would be endowed with a lower credit because
of the fall in the nominal interest rate. Hence, future governments would find it optimal to generate inflation, as we explained above. However, future expected inflation implies both a resource cost in the future and a current worsening of the inflation-output trade-off. Therefore, the current government decides to raise taxes so as to contain public deficits and future inflation. As a result, hours worked fall. Such a trade-off is absent when balanced budget is assumed.\textsuperscript{20} Finally, government expenditure is barely used for stabilization under either regime, as its responses closely resemble those under Pareto efficiency. If anything, public spending increases less and its volatility is sub-optimally low under RD in the attempt to contain public deficit.

All in all, the implications of fiscal discretion and Ramsey monetary policy are rather counterintuitive at a first glance. Lack of commitment forces the government to stabilize debt, which leads to inefficiently tight fiscal policies and failure to stabilize real economic activity following a shock. Ramsey monetary policy coupled with discretionary fiscal policy fails to stabilize inflation and output as would be the case under joint commitment – hence fiscal discretion hampers monetary commitment. (SHOULD WE CITE HERE LUISA’S PAPER, fiscal discretion destroys monetary commitment?). The trade-off between inflation and output stabilization is resolved in favor of the former for values of $\gamma$ etc etc. (we will maybe do sensitivity with respect to this, before editing the final version of this paragraph.)

5.3 Taylor-type rules and fiscal discretion

Assume that the monetary authority commits to a simple Taylor-type rule whereby the nominal interest rate responds to current inflation, in deviation from its steady-state $\pi$, according to:

$$\log(1 + i_t) = \phi_0 + \phi_{\pi} \log \left(\frac{\pi_t}{\pi}\right). \tag{28}$$

Parameters $\phi_0$, $\phi_{\pi}$ and $\pi$ are predetermined at the constitutional stage and they fully describe the monetary strategy $\sigma_m = \{i(\pi_t)\}_{t \geq 0}$. The plan prescribes an interest rate path that depends on fiscal policy only to extent that it affects the inflation rate. We think that this is an interesting case, even though it is a simple one, because an endogenous inflation response roughly agrees with the intentions declared by most of inflation targeting central banks. We keep all our previous assumptions about fiscal policy and we compute the outcome of the game for any given $\sigma_m$; fiscal policy is optimal in the sense defined in section 5.1. We find that the monetary policy strategy alters the way all successive fiscal policy makers interact among each other. As a result, rule (28) has an impact on steady-state debt. In particular, the level of debt is positive provided the inflation response of the nominal interest rate is large enough, i.e. $\phi_{\pi} > 1/\beta$. Furthermore, the tougher is the central bank in stabilizing inflation and the higher is the steady-state inflation rate, the lower is the steady-state level of debt.

Before turning to the formal analysis, it is useful to notice upfront that, despite its simplicity, the monetary rule in (28) implies complex strategic interaction between monetary and fiscal authorities. In fact, now the strategies of both the monetary and fiscal authorities are contingent, directly or indirectly, on the actions of their followers, whose

\textsuperscript{20}The case of wage mark-up shocks, analyzed in Gnocchi (2013) under the assumption of balanced budget, is not really interesting in our setup. This is because we have enough instruments to make trivial the stabilization trade-offs. Impulse responses after a wage mark-up shock in our setup are available upon request.
incentive can thus be manipulated by their respective leaders. These strategies are labelled as closed loop in dynamic game theory. Therefore, in the rest of the paper we use terms closed loop strategies and Taylor-type rules interchangeably. By prescribing a specific interest-rate response to current inflation, the monetary rule with \( \phi_\pi > 1/\beta \) raises the real interest rate with inflation. Hence the rule gives the fiscal authority the incentive not to raise inflation whenever she inherits a positive level of debt: by doing so, she would only worsen public accounts.

In the remainder of the section, we briefly describe how we compute the outcome of the game, we illustrate our main results and we finally conclude by comparing open and closed loop strategies.

We solve again the problem by adopting a primal approach whereby the fiscal authority decides on all endogenous variables, subject to the constraint that they must be consistent with a competitive equilibrium and with the Taylor-type rule. As before, the problem can be written recursively as

\[
W^{fT}_t = \max_{C_t, N_t, b_t, \eta_c, \tau_t, G_t, \eta_t} \left\{ (1-\gamma) \ln C_t + \gamma \ln G_t - \frac{(N_t)^{1+\phi}}{1+\phi} + \beta E_t W^{fT}_{t+1} \right\}
\] (29)
s.t.
\[
z_t N_t - C_t - G_t - \frac{\gamma}{2} (\pi_t - 1)^2 = 0;
\]
\[
\frac{1-\gamma}{C_t (1+i_t)} - \beta E_t \frac{1-\gamma}{C^T (b_t, s_{t+1}) \Pi^T (b_t, s_{t+1})} = 0;
\]
\[
\frac{b_t}{1+i_t} + \left( mc_t z_t - \frac{N_t^2 C_t}{1-\phi} \right) N_t - \frac{b_{t-1}}{\pi_t} - G_t = 0;
\]
\[
\beta E_t \frac{C_t \Pi^T (b_t, s_{t+1}) (\Pi^T (b_t, s_{t+1}) - 1)}{C^T (b_t, s_{t+1})} + \frac{\eta^p}{\gamma} z_t N_t \left( mc_t - \frac{\eta^p - 1}{\eta^p} \right) - \pi_t (\pi_t - 1) = 0;
\]
and
\[
\log (1+i_t) - \phi_0 - \phi_\pi \log \left( \frac{\pi_t}{\pi} \right) = 0,
\]
taking as given \( b_{t-1} \) and unknown functions \(^{22} C^T \) and \( \Pi^T \). In addition, we require (9) to hold. As in the previous section, we substitute for \( \tau_t \) into (19) from the household’s optimality condition (8). As before, the problem is recursive. Then, if an equilibrium exists, there must exist a value \( W^{fT}_t \) and time-invariant functions \( C^T \) and \( \Pi^T \) that satisfy all the conditions for a continuation competitive equilibrium and are consistent with fiscal optimality at \( h_{t+1,f} \).

As in the previous section, we first set \( z_t = 1 \) for all \( t \) and analyze the non-stochastic steady-state of the model, as a function of the inflation coefficient \( \phi_\pi \) and of the steady-state inflation rate \( \pi \). One can indeed show that if \( \phi_0 = \log (\pi/\beta) \), \( \pi \) is a steady-state. We stick to the case of \( \gamma = 20 \). Figure 5 shows debt-to-GDP ratio in percentage points, measured on the vertical axis as a function of coefficient \( \phi_\pi \), measured on the horizontal axis. We repeat the plot for different values of the annualized inflation rate, which is computed as \( \Pi = 4 (\pi - 1) \). It is evident that debt-to-GDP ratio decreases in both \( \phi_\pi \) and \( \Pi \). The result is entirely due to strategic interaction between monetary and

\[ ^{21} \]The lagrangian function and the first-order conditions associated to this maximization are reported in Appendix D. As above, we replace \( W^{fT}_t (s_t, b_{t-1}) \) with \( W^{fT}_t \).

\[ ^{22} \]Now we can express \( C^T \) and \( \Pi^T \) as functions of \( s_t \) and \( b_{t-1} \) only, because the nominal interest rate is chosen by the fiscal authority consistently with the monetary rule.
fiscal policymakers. To begin with, the interaction between each fiscal authority and her followers can be explained as in the case of monetary Ramsey strategies. At any point in time, say \( t \), the fiscal policymaker wants her successor to internalize that inflation in \( t + 1 \) is welfare detrimental from a time \( t \) perspective. In an environment where the nominal interest rate is not contingent on fiscal policy variables, leaving some positive assets to future authorities is the only viable discipline device. In fact, inflation affects the real service of debt through the government budget constraint. In particular, positive inflation is costly in terms of public accounts because it reduces the implicit real interest rate earned by the government given the nominal interest rate. However, with a Taylor-type rule, the nominal interest rate is not taken as given by the fiscal authorities as they internalize the monetary strategy \( \sigma_m \). It follows that the monetary rule may affect the incentives of fiscal authorities and, as a consequence, the way they interact among each other. For example, under the Taylor-type rule we assume, inflation affects the nominal interest rate. One can easily assess the impact of inflation on the government’s budget by computing the derivative of \( D_t \equiv b_t/(1+i_t) - b_{t-1}/\pi_t \) with respect to inflation, after internalizing the rule (28), and then evaluating it at the steady-state:

\[
D_\pi = \left. \frac{\partial D_t}{\partial \pi_t} \right|_{st} = -\frac{b(\beta \phi - 1)}{\pi^2}.
\]  

(30)

If \( \phi > 1/\beta \), an increase of the inflation rate worsens public accounts, i.e. \( D_\pi < 0 \), only if steady-state debt is positive. Therefore, each fiscal authority has now the incentive to leave her successor some positive debt in order to make her internalize the cost of future inflation (from current perspective). Moreover, this effect is higher in absolute value the

Figure 4: Steady-state debt under Taylor-type rules: the role of the inflation coefficient and of steady state inflation, \( \gamma = 20 \).
lower is the steady-state inflation rate. Hence, a positive inflation target reduces the level of debt needed as a discipline device, as compared to the zero inflation steady-state. We conclude that the endogenous monetary response to the inflation rate accounts for all these results. This is the so called sophisticated policy approach emphasized by Atkeson et al. (2010). One may wonder why specifying policies as a function of the history of exogenous events or as a feedback rule should make a difference. After all, by suitably substituting equilibrium outcomes into the feedback rule, it is always possible to obtain a function mapping a history \(s_t\) into the policy instrument. The intuition is straightforward. Closed loop strategies are in general more informative than an exogenous interest rate path, since they specify how policy would be conducted off-equilibrium. Therefore, if the followers behave strategically, the leader may engineer an off-equilibrium threat supporting an outcome that differs from the Ramsey case\(^{23}\). As a consequence, the steady-state level of debt has a simple interpretation: it is such that the incentive to produce deflation in order to improve the government’s budget constraint is exactly offset by the incentive to inflate in order to push real activity towards Pareto efficiency. The role played by off-equilibrium threats can be understood by considering a deviation from equilibrium on the part of a fiscal authority who starts playing at the non-stochastic steady-state. Should the fiscal authority deviate and generate a lower inflation rate than the actual outcome \(\pi\), the central bank would respond by lowering the nominal interest rate and thus improving \(\text{ceteris paribus}\) public accounts. Such a response would be costly for the government in charge. In fact, her followers would inherit a lower level of debt and find it appealing to inflate. If the cost is large enough to offset the benefit of deflation, the current fiscal authority does not deviate from equilibrium, \(b\) and \(\pi\) are steady-state outcomes and the off-equilibrium threat is not executed.

We conclude by briefly inspecting the dynamics after a positive technology shock. We set \(\phi_0 = -\log \beta\), i.e. consistently with a zero inflation steady-state\(^{24}\), and we let vary the inflation coefficients between 1.5 and 10000. The latter value well approximates a strict inflation targeting policy, \(\phi_{\pi} \to \infty\), where inflation is on target at all times. All variables are scaled and represented as in section 5.2. Economies are shocked at their respective non-stochastic steady-states. One can easily see that the effects of the monetary rule on fiscal responses are twofold. On the one hand, the Taylor-rule coefficient affects the dynamics of debt. When \(\phi_{\pi} \in [1.5, 4.5]\), the larger is the coefficient, the stronger is the (negative) response of real debt. Instead, when \(\phi_{\pi} \to \infty\) real debt is fully stabilized. Two opposite forces explain the result. Stronger monetary accommodation reduces the service of debt and thus public deficit. In other words, similarly to our steady-state argument, a higher Taylor rule coefficient improves public accounts to a greater extent whenever inflation falls. And inflation typically does fall after a transitory productivity shock for a finite \(\phi_{\pi}\). However, stronger monetary accommodation also reduces the equilibrium response of inflation to the shock. It follows that the response of debt is not monotone in \(\phi_{\pi}\): it first increases and then decreases. For \(\phi_{\pi}\) large enough, it converges to zero so that real debt is constant as well as inflation. The outcome is not surprising if one takes into account that strict inflation targeting restores flexible price fluctuations.\(^{25}\) On the

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\(^{23}\)In general, not simply different but also welfare improving. Unless one restricts the strategy space of the leader, as we do here. However, optimal monetary policy under closed loop strategies is beyond the scope of this paper and we leave it to future research.

\(^{24}\)Impulse responses under the assumption of a positive steady-state inflation rate are virtually indistinguishable from the ones we present here. Hence, we omit them for economy of space. However, they are available upon request.

\(^{25}\)Notice that we do not want to argue that such a policy is necessarily desirable. In a model with flexible
other hand, the use of tax rates and government expenditure can be rationalized by the sub-optimality of the monetary rule. It is well known that for finite values of $\phi_\pi$, the monetary policy stance is too tight as compared to FR: consumption and output do not increase as much as they should; hours worked fall; inflation is not stabilized. However, fiscal policy is still chosen optimally. Therefore, public spending can be used to sustain aggregate demand and tax rates to prevent inflation from falling.\footnote{Recall that in a baseline New-Keynesian model a positive exogenous shock to government expenditure increases both output and inflation; in contrast, a positive exogenous shock to income tax rates increases inflation and reduces output.} This conclusion is evident by comparing the response of fiscal variables across different values of $\phi_\pi$. The increase in government expenditure is inversely proportional to $\phi_\pi$ and it coincides with FR under strict inflation targeting. Also, taxes and hours are perfectly smooth only when prices are fully stabilized. Hence, we conclude that taxes and government expenditure are used to cope with the monetary sub-optimality induced by the simple Taylor-type rule.

6 Conclusions

TBW

prices, inflation would be the right instrument to use in order to make real debt state-contingent and thus constant in the presence of stationary shocks. However, in our model with a strict inflation target, the reason why debt is constant is the off-equilibrium threat extensively discussed above. Hence, very small errors in the measurement of whatever variable relevant for the central bank’s behavior would induce abrupt fluctuations in the real side of the economy, making the policy likely sub-optimal.
References


Appendix

A Full Ramsey

The first-order conditions relative to the variables $C_t, N_t, G_t, \pi_t, mc_t, b_t, i_t$ are:

FOC $C_t$: \[
\frac{1 - \chi}{C_t} - \lambda_t^f + \lambda_t^b \frac{1 - \chi}{(1 + i_t)C_t^2} - \lambda_t^{b-1} \frac{1 - \chi}{\pi_t C_t^2} - \lambda_t^{N_t \gamma^0} \frac{N_t^{1+\gamma}}{1 - \chi} \Phi \exp \mu_t^w + \lambda_t^{\chi^0} \Phi \exp \mu_t^w + (31) \]

FOC $N_t$: \[
- N_t^\gamma + z_t \lambda_t^f + \lambda_t^s \left[ mc_t z_t - (1 + \varphi) \frac{N_t^\gamma C_t \Phi \exp \mu_t^w}{1 - \chi} \right] + \lambda_t^{\eta^p} \frac{\eta^p - 1}{\eta^p} \gamma z_t = 0 \] \[
(32) \]

FOC $G_t$: \[
\frac{X}{G_t} - \lambda_t^f - \lambda_t^s = 0, \]

FOC $\pi_t$: \[
- \lambda_t^f (\pi_t - 1) - \lambda_t^b \frac{1 - \chi}{\pi_t^2 C_t} + \lambda_t^{b-1} \frac{1 - \chi}{\pi_t^2 C_t} - \lambda_t^p (2\pi_t - 1) + \lambda_t^{\lambda_t^s} \frac{C_t^{-1}}{C_t} (2\pi_t - 1) = 0, \]

FOC $mc_t$: \[
\lambda_t^s N_t z_t - \lambda_t^p N_t z_t \frac{\eta^p}{\gamma} = 0, \]

FOC $b_t$: \[
\frac{\lambda_t^s}{1 + i_t} - \frac{\lambda_t^{s+1} \beta}{\pi_t^{s+1}} = 0, \]

FOC $i_t$: \[
\frac{\lambda_t^s (1 - \chi)}{C_t (1 + i_t)^2} - \frac{\lambda_t^s b_t}{(1 + i_t)^2} = 0, \]

Equations (31) to (37) together with the first-order conditions relative to the lagrangian multipliers $\lambda_t^f, \lambda_t^b, \lambda_t^s, \lambda_t^p$ form a system of eleven equations in eleven variables.

B Robustness

C Fiscal discretion

The lagrangian associated to the fiscal policy problem is

\[
L_t^f (b_{t-1}, s_t) = \left[ (1 - \chi) \ln C_t (i) + \chi \ln G_t - \frac{(N_t (i)))^{1+\gamma}}{1 + \varphi} \right] + \beta E_t L_t^f (b_t, s_{t+1}) + \lambda_t^f \left[ z_t N_t - C_t - G_t - \frac{1}{2} (\pi_t - 1)^2 \right] + \lambda_t^b \left[ \frac{1 - \chi}{C_t (1 + i_t)} - \beta E_t \frac{1 - \chi}{C (b_t, s_{t+1} \Pi (b_t, s_{t+1}))} \right] + \lambda_t^s \left[ \frac{b_t}{1 + i_t} + \pi_t w_t N_t - \frac{b_{t-1}}{\pi_t} - G_t - \gamma \right] + \lambda_t^p \left[ \beta E_t \frac{C_t}{C (b_t, s_{t+1} \Pi (b_t, s_{t+1}))} (\Pi (b_t, s_{t+1}) - 1) + \frac{\eta^p}{\gamma} z_t N_t \left[ mc_t - \frac{\eta^p - 1}{\eta^p} \right] - \pi_t (\pi_t - 1) \right] \]
\( \tilde{\lambda}_t \) is the lagrangian multiplier for the fiscal problem on the resource constraint, \( \tilde{\lambda}_t^p \) is the lagrangian multiplier on the euler equation of the households, \( \tilde{\lambda}_t^s \) is the lagrangian multiplier on the government budget constraint, and \( \tilde{\lambda}_t^p \) is the lagrangian multiplier on the Phillips curve. The first-order conditions of the fiscal policymaker relative to \( C_t, N_t, G_t, \pi_t, mc_t, b_t \) are:

\[
\frac{1 - \chi}{C_t} + \tilde{\lambda}_t^s \frac{N_t^{1+\varphi} \Phi exp\{\mu_t^w\}}{1 - \chi} - \frac{\tilde{\lambda}_t^b}{C_t^2 (1 + i_t)} + \tilde{\lambda}_t^p E_t \left[ \beta \frac{\pi_{t+1}(\pi_{t+1} - 1)}{C_{t+1}} \right] = \tilde{\lambda}_t^f \tag{39}
\]

\[
\tilde{\lambda}_t^f z_t + \tilde{\lambda}_t^s m_c z_t - (1 + \varphi) \frac{\tilde{\lambda}_t^s N_t^s C_t \Phi exp\{\mu_t^w\}}{1 - \chi} - \tilde{\lambda}_t^p \frac{\eta^p}{\gamma} \left( \eta^p \frac{\pi_{t+1} - 1}{\eta^p} - mc_t \right) = N_t^p \tag{40}
\]

\[
\tilde{\lambda}_t^f = \frac{\chi}{G_t} - \tilde{\lambda}_t^s \tag{41}
\]

\[
\tilde{\lambda}_t^p = \frac{\tilde{\lambda}_t^s \gamma}{\eta^p} \tag{43}
\]

\[
\beta E_t \frac{\partial L_{t+1}}{\partial b_{t}} + \frac{\tilde{\lambda}_t^s}{1 + i_t} + \tilde{\lambda}_t^p C_t E_t \left[ \frac{\partial \Pi}{\partial b_t} \frac{2\pi_{t+1} - 1}{C_{t+1}} - \frac{\partial C}{\partial b_t} \frac{\pi_{t+1}(\pi_{t+1} - 1)}{C_{t+1}^2} \right] + \frac{\tilde{\lambda}_t^b E_t}{\beta C_t} \left[ \frac{\partial C}{\partial b_t} \frac{1}{C_{t+1} \pi_{t+1}} + \frac{\partial \Pi}{\partial b_t} \frac{1}{C_{t+1}^2 \pi_{t+1}} \right] = 0 \tag{44}
\]

\[
\frac{\partial L_t}{\partial b_{t-1}} = -\frac{\tilde{\lambda}_t^s}{\pi_t} \tag{45}
\]

Rearranging equations (39) through (45) yields the system of four equations

\[
\frac{1 - \chi}{C_t} + \tilde{\lambda}_t^s \left[ \frac{1 - N_t^{1+\varphi} \Phi exp\{\mu_t^w\}}{1 - \chi} + \beta \frac{\gamma}{\eta^p} \frac{\pi_{t+1}(\pi_{t+1} - 1)}{C_{t+1}} \right] - \frac{\tilde{\lambda}_t^b}{C_t^2 (1 + i_t)} = \frac{\chi}{G_t} \tag{46}
\]

\[
\frac{\tilde{\lambda}_t^s}{1 + i_t} \frac{\chi}{G_t} - \frac{\tilde{\lambda}_t^s z_t}{\eta^p} - (1 + \varphi) \frac{\tilde{\lambda}_t^s N_t^s C_t \Phi exp\{\mu_t^w\}}{1 - \chi} = N_t^p \tag{47}
\]

\[
\tilde{\lambda}_t^s \left[ \gamma(\pi_t - 1) + \frac{b_{t-1}}{\pi_t} + \frac{\gamma}{\eta^p} (2\pi_t - 1) \right] = \gamma(\pi_t - 1) \frac{\chi}{G_t} \tag{48}
\]

\[
\frac{\tilde{\lambda}_t^s}{1 + i_t} - \frac{\tilde{\lambda}_t^s \gamma}{\eta^p} \beta C_t E_t \left[ \frac{\partial \Pi}{\partial b_t} \frac{2\pi_{t+1} - 1}{C_{t+1}} - \frac{\partial C}{\partial b_t} \frac{\pi_{t+1}(\pi_{t+1} - 1)}{C_{t+1}^2} \right] + \frac{\tilde{\lambda}_t^b \beta E_t}{\beta C_t} \left[ \frac{\partial C}{\partial b_t} \frac{1}{C_{t+1} \pi_{t+1}} + \frac{\partial \Pi}{\partial b_t} \frac{1}{C_{t+1}^2 \pi_{t+1}} \right] = \beta E_t \frac{\tilde{\lambda}_t^s}{\pi_{t+1}} \tag{49}
\]

while equations (41) and (43) serve the only purpose of retrieving the lagrangian multipliers \( \tilde{\lambda}_t^f \) and \( \tilde{\lambda}_t^p \).
The problem of the monetary authority when fiscal policy is discretionary is given by:

$$\mathcal{L}^M = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ (1 - \chi) \ln C_t + \chi \ln G_t - \frac{(N_t)^{1+\varphi}}{1 + \varphi} \right] + \lambda_t^b \left[ \frac{1 - \chi}{C_t(1 + i_t)} + \frac{1 - \chi}{C_t \pi_t} \right] + \lambda_{t-1}^b \left[ \frac{1 - \chi}{C_t(1 + i_{t-1})} + \frac{1 - \chi}{C_t \pi_{t-1}} \right] \right\}$$

(50)

$$+ \lambda_t^s \left[ \frac{b_t}{1 + i_t} + \tau_t N_t - \frac{b_t}{\pi_t} - G_t - \tau^p \right]$$

$$+ \beta \lambda_{t+1}^s \left[ \frac{b_{t+1}}{1 + i_{t+1}} + \tau_{t+1} w_{t+1} N_{t+1} - \frac{b_t}{\pi_{t+1}} - G_{t+1} - \tau^p \right] +$$

$$+ \lambda_t^p \left[ \frac{\beta C_t}{C_t \pi_t} (\pi_t - 1) + \frac{\eta^p}{\gamma} \tau_t N_t \left( m c_t - \frac{\eta^p - 1}{\eta^p} \right) - \pi_t (\pi_t - 1) \right]$$

$$+ \lambda_{t-1}^p \left[ \frac{\beta C_t}{C_t \pi_{t-1}} (\pi_{t-1} - 1) + \frac{\eta^p}{\gamma} \tau_{t-1} N_{t-1} \left( m c_{t-1} - \frac{\eta^p - 1}{\eta^p} \right) - \pi_{t-1} (\pi_{t-1} - 1) \right]$$

$$+ \lambda_t^{d1} \left[ \frac{1 - \chi}{C_t} + \lambda_t^s \left[ 1 - \frac{N_t^{1+\varphi} \Phi \exp \{ \mu_t^w \}}{1 - \chi} + \beta \frac{\gamma}{\eta^p} \pi_t (\pi_t - 1) \right] - \frac{\lambda_{t-1}^b}{C_t^2 (1 + i_{t-1})} - \frac{\chi}{G_t} \right]$$

$$+ \lambda_{t-1}^{d1} \left[ \frac{1 - \chi}{C_{t-1}} + \lambda_{t-1}^s \left[ 1 - \frac{N_t^{1+\varphi} \Phi \exp \{ \mu_{t-1}^w \}}{1 - \chi} + \beta \frac{\gamma}{\eta^p} \pi_{t-1} (\pi_{t-1} - 1) \right] - \frac{\lambda_{t-1}^b}{C_{t-1}^2 (1 + i_{t-1})} - \frac{\chi}{G_{t-1}} \right]$$

$$+ \lambda_t^{d2} \left[ \frac{z_t}{G_t} - \frac{\lambda_t^s}{\eta^p} - \frac{(1 + \varphi) \tau_t N_t C_t \Phi \exp \{ \mu_t^w \}}{1 - \chi} - \frac{N_t^\varphi}{1 - \chi} \right]$$

$$+ \lambda_t^{d3} \left[ \lambda_t^s \gamma (\pi_t - 1) + \frac{b_t}{\pi_t} - \frac{\gamma}{\eta^p} (2\pi_t - 1) \right] - \frac{\lambda_{t+1}^s}{\pi_t} \frac{\chi}{G_t}$$

$$+ \beta \lambda_{t+1}^{d3} \left[ \frac{\lambda_{t+1}^s}{\pi_{t+1}} \gamma (\pi_{t+1} - 1) + \frac{b_t}{\pi_{t+1}} - \frac{\gamma}{\eta^p} (2\pi_{t+1} - 1) \right] - \frac{\lambda_{t+1}^s}{\pi_{t+1}} \frac{\chi}{G_{t+1}}$$

$$+ \lambda_t^{d4} \left[ \frac{\lambda_t}{1 + i_t} - \frac{\lambda_t^s}{\eta^p} \beta C_t E_t \left[ \frac{\partial \Pi}{\partial b_t} \frac{2\pi_t - 1}{C_{t+1}} - \frac{\partial \mathcal{C}}{\partial b_t} \pi_{t+1} (\pi_{t+1} - 1) \right] \right]$$

$$+ \lambda_{t-1}^{d4} \left[ \frac{\lambda_{t-1}}{1 + i_{t-1}} - \frac{\lambda_{t-1}^s}{\eta^p} \beta C_{t-1} \left[ \frac{\partial \Pi}{\partial b_{t-1}} \frac{2\pi_{t-1} - 1}{C_t} - \frac{\partial \mathcal{C}}{\partial b_{t-1}} \pi_{t-1} (\pi_{t-1} - 1) \right] \right]$$

$$+ \lambda_t^{d4} \left[ \frac{\lambda_t^b}{\beta} \left[ \frac{\partial \mathcal{C}}{\partial b_t} \frac{1}{C_t^2 \pi_t} + \frac{\partial \Pi}{\partial b_t} \frac{1}{C_t \pi_t^2} \right] - \beta \frac{\lambda_t^s}{\pi_t} \right] \right\},$$

where $\tilde{\lambda}$s are the lagrangian multipliers on the constraints in the monetary policy problem and the $\hat{\lambda}$s are the lagrangian multipliers on the constraints in the fiscal policy problem. The expressions $\partial \mathcal{C} / \partial b_t$ and $\partial \Pi / \partial b_t$ denote the derivative of the private agent’s rule for consumption and inflation, respectively, relative to the level of debt.
D Fiscal discretion with Taylor-type rules

\[
\mathcal{L}^f_t(b_{t-1}, s_t) = \left[ (1 - \chi) \ln C_t(i) + \chi \ln G_t - \frac{(N_t(i))^{1+\varphi}}{1 + \varphi} \right] \\
+ \bar{\lambda}_f \left[ z_t N_t - C_t - G_t - \frac{\gamma}{2} (\pi_t - 1)^2 \right] + \bar{\lambda}_b \frac{1 - \chi}{C_t(1 + i_t)} - \beta E_t \frac{1 - \chi}{C(b_t, s_{t+1}) \Pi(b_t, s_{t+1})} \\
+ \bar{\lambda}_s \left[ \frac{b_t}{1 + i_t} + \tau_t w_t N_t - \frac{b_{t-1}}{\pi_t} - G_t - \tau^p \right] + \bar{\lambda}_p \left[ \beta E_t \frac{C_t}{C(b_t, s_{t+1}) \Pi(b_t, s_{t+1})} (\Pi(b_t, s_{t+1}) - 1) \\
+ \frac{\eta_p}{\gamma} z_t N_t \left( mc_t - \frac{\eta_p - 1}{\eta_p} \right) - \pi_t (\pi_t - 1) \right] \\
+ \bar{\lambda}_i \left[ \log \left( \frac{i_t}{i} \right) + \phi_p i \log \left( \frac{\pi_t}{\pi} \right) \right] + \beta E_t \mathcal{L}^f_t(b_t, s_{t+1}),
\]

where \( \lambda_i \) is the lagrangian multiplier on the Taylor-type rule followed by the monetary authority. The fiscal authority maximizes (51) relative to \( C_t, N_t, G_t, \pi_t, mc_t, i_t, b_t \).